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BERTRAND RUSSELL'S CONSTRUCTION OF THE EXTERNAL WORLD

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BERTRAND RUSSELL'S CONSTRUCTION OF THE EXTERNAL WORLD

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CHAPTER I

THE DEVELOPMENT OF BERTRAND RUSSELL'S PHILOSOPHY

THE work of Bertrand Russell in contemporary philosophy has covered a wide variety of topics, and has attempted to answer many of the problems traditionally associated with philosophy. He has made important contributions to several fields, both in specific results and in new and suggestive hypotheses. The influence of his philosophy is apparent upon any examination of contemporary philosophy, especially in logic, epistemology, logical analysis, and the philosophy of science. However, the very fact that his philosophical interests have covered a wide range makes it difficult to obtain a brief systematic account which will do justice to them.

Several recurrent strains of considerable importance can, however, be found in his writings. I believe it is possible to give a brief account of these aspects that will still present the relevant issues involved and even permit some evaluation of Russell's conclusions. In his later work we find that Russell is concerned with a problem that has been of great importance to philosophers, traditional and contemporary, the problem of the basis of our knowledge of the external world, both as it is known to common sense and to science. Russell's own solution of this problem underwent development as he reconsidered his philosophy; for he is an example of a philosopher who does not maintain that his answer is the final truth. In the study of this problem he found he could utilize certain methodological principles, especially the employment of 'logical constructions', which he had developed in his earlier, more specialized, investigations. In these earlier writings, Russell made important contributions to the foundations of mathematics and symbolic

logic, of such importance that if he had done no further work those contributions would themselves have given him a secure place among contemporary philosophers.

What I shall emphasize in Russell's work, then, is his analysis of the problem of our knowledge of the external world, and the use of constructions in that analysis. The importance of his earlier technical writings not only as solutions to particular problems but as gradually making clear a new principle by which more general philosophical problems could be attacked is a development which Russell himself presents in discussing the growth of his philosophical views.[1] In the following pages I shall retrace at some length this development.

Russell's early interest in mathematics and logic led to two of his important works, the *Principles of Mathematics* (1903) and *Principia Mathematica* (with Whitehead's collaboration, 1910–13). He was concerned with the nature of mathematical knowledge, with that abstract, formal quality that is referred to when mathematics is said to be 'necessary' or *a priori*. Russell was not convinced that the solutions found, e.g. by Hume or Kant, were valid; and he looked for some other explanation. Further, certain mathematical notions, especially those of the nature of number, infinity, and continuity, he felt had never adequately been explained, and attempted explanations had led to philosophic theories—such as Kant's account of space, which Russell felt he could not accept. The general philosophic view which Russell had at this time was a rather extreme realism, believing in the reality of many kinds of entities, especially physical objects, minds and universals. He used in his mathematical investigations also the work of preceding mathematicians such as Cantor, Dedekind and Weierstrass, and more important, the discoveries made in

[1] Russell gives brief accounts of the development of his philosophical views in 'Logical Atomism', in *Contemporary British Philosophy*, first series, New York and London, 1924, and in 'My Mental Development', in *The Philosophy of Bertrand Russell*, Evanston and Chicago, 1944. A discussion from the same point of view (but without personal reference) will be found in his *A History of Western Philosophy*, New York, 1945, pp. 828–34.

symbolic logic by the Italian school. He found that by the help of the new logic it was possible to answer the mathematical problems he was investigating. The question of the nature of mathematical knowledge was answered, or at least shown to be another problem, by finding that it was possible to 'identify' mathematics and logic. He showed that the propositions of pure mathematics could be derived from those of logic, and consequently the 'necessity' and 'truth' of mathematics were the same as those of logic. The further problem important for his later philosophy that he found he could answer was the nature of 'number'. Previously, on the basis of his realistic philosophy, it had been necessary to assume that numbers were real entities, but employing a procedure which he later came to call 'logical constructions' he was able to show that numbers do not refer to real entities 'numbers' but to classes of classes, entities already known.

A few years later in *Principia Mathematica* he improved the logical apparatus and was able to give these mathematical notions symbolic formulation. By this time he had discovered also that he was able to analyse the notion of 'classes' and 'descriptions' as he had done with 'number'. The necessity for such an analysis again was derived in part from his philosophical realism, for without such an analysis these terms would refer to entities which would be 'classes' or 'objects described'. The theory of descriptions was especially important for his later work. An object described is what is referred to in such statements as 'the cat was outside last night', or 'the 4th President of the United States was bald', or 'the round-square is square'. Following a realistic analysis of propositions, it seemed to some philosophers that descriptive phrases referred to entities of some kind, such as 'cat', or '4th President of the United States', or 'round-square'. However, the notion of these entities involves certain difficulties, for although objects described which do not exist, such as '4th President of the United States', might be said to subsist, what shall we do with self-contradictory objects, such as 'round-square'? Russell found

that again the use of logical constructions provided a successful solution to the difficulty. It was possible by their means to analyse descriptive statements so that they are found to refer to properties, or concepts, whose metaphysical status offered no difficulty to Russell. The 'object described' disappears, along with its questionable reality.

The success of the employment of logical constructions in finding solutions to mathematical and logical problems encouraged Russell to give them wider application in attacking more general philosophical problems. But what are general philosophical problems? Russell believes that such problems will be disclosed through a critical examination of ordinary common-sense knowledge and more complex scientific knowledge. The simplest way to proceed may be to follow Descartes' procedure (to which Russell often refers) by examining our knowledge to see if it contains an element of doubt, and, if so, to analyse that knowledge to see what elements in it we can be most sure of, and try to justify the remainder of our knowledge on the basis of this more certain knowledge. His own reference to Descartes is enlightening I think in understanding the point of view from which he is beginning. For both, there is the same problem of determining what are the rational or logical grounds for believing our knowledge, and to what extent our knowledge can seem rationally indubitable. The task of logical analysis is to find what is the most certain of our knowledge; once this basis has been found we may start a more constructive task, that of rebuilding the remainder of our knowledge on this certain basis.

The world that Russell believed in in the *Principles of Mathematics* he was not too concerned with criticizing. It assumed the existence of external objects, of minds, and of universals. But the process of doubting gradually led him to extensively modify such a picture. Can we assume that physical objects exist in the sense that common sense believes that they do? In the *Principles of Mathematics* they exist without question and we know them directly, in *The Problems of Philosophy* (1912) they exist, but now

we can only know them through our own sensations (sense-data). The objects that we know most directly in empirical knowledge are sense-data, and we infer the existence of the persistent, permanent physical object from them. We can infer little as to their real nature, but Russell feels that we can be sure that they do exist, cause our sense-data, and constitute what we consider the real physical world. But further reflection shows that even this belief in material objects is uncertain, and in *Our Knowledge of the External World* (1914) Russell finds that about the only beliefs which cannot be easily doubted are those in our own perceptual experience.

The existence of material objects becomes a problem, as Russell conceived it at the time of *Our Knowledge of the External World*, through critical reflection upon ordinary empirical knowledge. Ordinary common-sense views of perceptual knowledge are that there are objects, independent of ourselves, which we perceive by means of our sense-organs. These objects may or may not 'resemble' our perceptions, but there is sufficient correspondence between them of some kind so that our perceptions are at least a good practical guide in dealing with those objects. The independence of material objects is shown, at this stage of thought at least, by the fact that the actions of different persons are related to a group of objects that concerns them all; they can communicate about a common subject-matter, they can transform objects common to all of them, and can predict the future movements, and their reactions to them, of the common group of objects. However, the usual arguments showing the impermanence of perceptions, their changes with any change in the observer or the intervening medium, show that our perceptions depend, in part at least, upon ourselves, and lead Russell to believe that the material we use in perceptual knowledge is not knowledge of physical objects, but individual perceptions. At the time of *Our Knowledge of the External World* Russell maintained an epistemology that our perceptual knowledge has for its basic material individual perceptions; permanent material objects (if they exist)

would have to be known in some way by means of our individual perceptions, or as Russell says, would be 'inferences' from them. Such an inference is made when one argues that there are independent physical objects for any of the reasons I suggested above. The problem with common-sense perception, Russell believes, is then to find on what grounds the inference from our individual perceptions to permanent material things can be validated (justified), or, the better solution which he thinks he has obtained, to find a way to account for all the functions permanent material objects satisfy without having to assume that such objects exist.

Logical constructions, which Russell had developed in solving some of his earlier problems, were the means by which he believed a solution could be obtained here as well. The common-sense belief in material objects, Russell maintains, is an inference based on the individual's perceptions, but it is an inference to entities that can never be observed. If some way could be found by which perceptions themselves, or certain groups of them, could serve all the purposes for which material objects are usually employed, then the inference to those objects could be dispensed with and we could rest secure with our knowledge of percepts. This function, Russell believes, the construction of material objects serves.

The criticism of ordinary empirical knowledge to show that it is based on our own perceptions, and that permanent physical objects are known, if at all, only by means of these perceptions, has removed the world of permanent substances from our immediate experience. How much further must the world of physics be removed! 'The world of physics', as Russell calls it, is even more removed from our perceptions than the world of common-sense objects, since physics deals with the 'matter' of which those objects are composed and the space and time within which they exist. Scientific entities do not resemble our perceptions, the colours, sounds, shapes we perceive do not have the same characteristics that the waves, vibrations, or particles in space possess that produce those perceptions.

'Matter' has gradually become more abstract from the primitive elements of the Greeks to the atoms of modern theory, to protons and electrons, to the 'matter' of modern physics. Space and time have become more complex as physics progressed from Newtonian conceptions to those of our time. How can we 'bridge the gulf between physics and perception',[1] how can we relate the description of the world given by physics to the world as it appears in our perceptions?

The relation between physics and perception is more important to Russell than simply for the sake of curiosity finding the meaning of physical terms in terms of our perceptions. Determining that relation is necessary in finding an 'interpretation' for physics. An 'interpretation' of physics, like that of any other abstract system, is finding a procedure to correlate the entities of the abstract system with the empirical world. In physics, an interpretation is necessary in applying it to the world and achieving practical results in new inventions, engineering, and the like. Mechanics, for example, formulates laws relating bodies considered as masses situated at points in space, as particles without extension located in space, with spatial relations to other such bodies without extension, and acted upon by forces moving them or putting them into equilibrium with other bodies. The temporal changes that take place are reckoned as happening at some 'instant' or between several instants, an instant being a location in time, a date, but having no duration—as a point marks a location in space but has no extension. Although physics has developed tremendously since mechanics was developed, mechanics is still applicable to the large perceptual objects of the world, it can be used to make valid inferences about the behaviour of these bodies. Such laws tell us how to build houses, how to shoot guns, what force we must use to start a train or a car; but all of these laws are stated in terms of masses concentrated at points in space and acting at instants of time. An 'interpretation' of these laws will give a way in which the laws can be translated to apply to bodies

[1] *The Analysis of Matter*, New York and London, 1927, p. 7.

with shapes and sizes, where distances are always between extended bodies and can never be given with the precision of distances between points, and where events in time always have durations and cannot be considered as happening at an instant.

Besides the practical use that an interpretation of physics has, an even more important requirement for one, Russell believes, is for the empirical verification of physics. If physics is to be empirically verified, there should be some procedure by which physical laws are related to observable events, so that events can serve as confirmation of laws. As Russell believes that the ultimate basis of our empirical knowledge is our own perceptions, physics to be verified would have to be interpreted ultimately in terms of our own perceptions.

> The laws of physics are believed to be at least approximately true, although they are not logically necessary; the evidence for them is empirical. All empirical evidence consists, in the last analysis, of perceptions; thus the world of physics must be, in some sense, continuous with the world of our perceptions, since it is the latter which supplies the evidence for the laws of physics.[1]

Russell has a favourite illustration which he uses to show the difficulty of the relation of physical and perceptual entities:[2] Physics tells us that the sun is 93 million miles away, that electromagnetic waves leaving the sun travel for 8 minutes, then affect the retina, the optic nerve, and finally produce in the brain the image of the sun that has led us to believe in, and has been the ultimate verification of, the existence of the optic nerve, the retina, the electromagnetic waves, the 93 million miles, and the sun as described by physics situated at that distance. The physical series of events 'causes' our image of the sun, but it is that image, which is believed to be the end result of the causal chain, that is the origin and justification of our belief in the causal chain. 'It is this curious oppositeness

[1] *The Analysis of Matter*, New York and London, 1927, p. 6.

[2] See, e.g. the statements in *Mysticism and Logic*, New York, 1918, pp. 134–5; *An Inquiry into Meaning and Truth*, New York and London, 1940, p. 146; *Human Knowledge*, New York, 1948, p. 8.

of direction between the order of causation as affirmed by physics, and the order of evidence as revealed by theory of knowledge, that causes the most serious perplexities in regard to the nature of physical reality.'[1]

At times Russell also discusses these problems in terms of which is the 'real' world, which the world of 'appearances'. Thus are our perceptions 'appearances', while the entities of physics and material objects are 'real', or are our perceptions really 'real'?[2] I do not think that a discussion from the point of view of appearance and reality makes any significant difference to the problem as I have already sketched it; although Russell does in some places discuss his problem in terms of 'appearance and reality', the specific problems which he proceeds to investigate are those I have mentioned. Consequently I shall seldom refer to the problem of 'appearance and reality' in this book. Russell's discussions will make sufficiently clear what his solution to this problem is.

It is then necessary, Russell believes, to find the nature of the entities used in physics. Like material objects, he maintains that they too can be constructed from perceptions, which makes it unnecessary for us to 'infer' their existence from our perceptions, but yet preserves whatever useful function they serve. The 'construction' of material objects and the entities of physics apparently leaves us with a metaphysical view of the world in which such entities are absent, one in fact which views the world as composed largely of percepts. Such a view can be found in *Our Knowledge of the External World* and other of Russell's works of the same period. Later, however, from the time of *The Analysis of Mind* (1921) and *The Analysis of Matter* (1927), although material objects and the entities of physics are still constructions, events independent of the observer are included in the construction and the world takes on a more

[1] *Mysticism and Logic*, p. 135.

[2] *Our Knowledge of the External World*, Chicago and London, 1914, Chapters III and IV; *Mysticism and Logic*, Chapter VII; *The Problems of Philosophy*, New York and London, 1912, Chapter I.

usual metaphysical appearance. Russell's metaphysics becomes, until recently at least, a form of 'neutral monism', in which there is one substance, events, and different arrangements of events, or different constructions from them, constitute 'mind' and 'matter'.

His own achievements in philosophy, Russell hopes, whatever lasting value they may have may at least serve to advance a new method in philosophy. Russell is only one of a long tradition of those who have envied the precise results of physical science and wished that philosophy could emulate them. Too often philosophy leads to only inconclusive results; speculative systems succeed each other, the more recent overthrowing its predecessor, and in its turn to be overthrown by a later. There seems to many no hope of finding an all-embracing view of man and the universe which will have any final validity. The objections to philosophy on the ground of its apparent lack of progress are familiar, and are expressions of such discouragement. Russell feels that it is necessary to make philosophy 'scientific', to let it secure answers to possibly small problems, but at least answers which seem well grounded, and on the basis of such answers proceed to larger generalizations. Something, at least, will have been definitely established. It is such a faith which Russell presumably feels that he is illustrating in his own work. Thus he calls his philosophy 'scientific' in the general sense of the word, that it is a cumulative process, finding definite answers to specialized questions, and only on this basis attempting to generalize.

His own philosophy, Russell feels, is also 'scientific' in a further sense. It uses a specialized tool that has been recently developed to arrive at answers for problems never before capable of a precise answer; that tool is mathematical logic. It is mathematical logic that aids Russell in his analysis of mathematics, of descriptions, and in the formation of constructions generally. It is this logic which also suggests possibilities in the analysis of many other problems. The importance of logic, the importance of the new philosophic method, are

subjects that Russell often discusses. He speaks of the 'logical-analytic' method in philosophy, even to characterize 'logic as the essence of philosophy'. He also develops the philosophy of 'logical atomism' and in his later works emphasizes the importance of 'logical analysis'.

In the remainder of this book I shall develop in some detail the brief account of Russell's philosophy that I have outlined in the preceding pages. First, I shall discuss the logical and mathematical problems for whose solution Russell first developed the use of constructions; then present the problem which he finds in our knowledge of material objects or scientific knowledge. It is not such a simple problem to find, partly because Russell himself is not as clear as one might wish, and further, because the exact nature of the problem itself is of some debate in epistemological writings. After attempting to clarify his problem, I shall present Russell's proposed solution and determine to what extent it can serve as an adequate analysis. I shall then discuss briefly the extent to which his 'new method' in philosophy can actually serve as a new and more productive method, and, finally, briefly survey his conclusions to determine at what points it is possible to improve his analysis.

CHAPTER II

THE CONSTRUCTION OF NUMBERS, DESCRIPTIONS, AND CLASSES

Those familiar with Russell's logical and mathematical work know the importance of his definitions of numbers, descriptions, and classes by means of logical constructions. It was his aim to show that pure mathematics could be derived from logic, and in so doing it was essential to define numbers in purely logical terms. This the construction of numbers made possible. In the first formulation of his system, however, there were certain difficulties caused by the notion of 'class'; his analysis of descriptions led to a definition of 'class' as a logical construction, an analysis which enabled Russell to avoid the earlier difficulties. It is my purpose here to discuss these definitions as solutions to separate problems, rather than to consider their place in the final development of his logical system. I shall discuss the problems Russell found in defining each of these concepts, and the analysis by means of constructions which he gives of each. The procedure which he devised in the solution of the logical and mathematical problems here is the one he later was to find of such usefulness in attacking more general philosophical problems.

PART I

NUMBERS

The derivation of pure mathematics from logic is found in its first extended formulation in the *Principles of Mathematics* (1903), but was given (with Whitehead's collaboration) a more rigorous and precise development in symbolic form in *Principia*

Mathematica (1910–13).[1] The system which Russell developed made it possible from a few undefined logical notions and primitive propositions to derive, not only propositions of logic, but the basic notions and further propositions of pure mathematics. Such a derivation provided an answer to the question of the nature of mathematical knowledge. It was no longer necessary to attempt an explanation of the apparent necessity and *a priori* character of mathematical statements on Kantian grounds, nor to try to base them on empirical evidence, such as Mill had done. Since mathematical propositions are part of logic, their nature is the same as that of logic, and while the nature of logical knowledge is not thereby determined, at least we have resolved doubts about the nature of mathematics.

In reducing pure mathematics to logic it is not sufficient that such mathematical entities as numbers be introduced without an analysis of them in logical terms. If they were merely introduced as undefined terms, a deductive system containing the propositions of pure mathematics could be constructed beginning from these basic terms and primitive propositions concerning them. This system would illustrate that mathematics can be organized as a deductive system and would be 'logical' in the sense that any deductive system is an example of the application of logical principles; but it would

[1] Russell's major works concerning the topics discussed in this Part are the *Principles of Mathematics*, Cambridge, 1903, *Principia Mathematica*, (with A. N. Whitehead), Cambridge, 1910–13, and the *Introduction to Mathematical Philosophy*, London and New York, 1919. The latter work is a clear, non-technical account of his chief logical and mathematical doctrines. A clear, simplified account of many of the points of Russell's system in the *Principles of Mathematics* can be found in L. Couturat, *Les Principes des Mathematiques*, Paris, 1905. The reader may find of interest briefer discussions in R. Carnap, *Foundations of Logic and Mathematics*, Chicago, 1939, especially Sections 14, 17, 20; C. G. Hempel, 'On the Nature of Mathematical Truth' (1945), reprinted in Feigl and Sellars' *Readings in Philosophical Analysis*, New York, 1949.

In *The Philosophy of Bertrand Russell*, edited by P. A. Schilpp, Evanston and Chicago, 1944, Reichenbach gives a generally favourable discussion of certain points of Russell's logic, including his definition of number, pp. 23 ff.

be of little assistance in determining the essential nature of mathematical knowledge. Russell's accomplishment is to show that much more can be achieved, that mathematics can be a part of logic in requiring the employment of no peculiarly mathematical entities, that the basic entities of mathematics can be defined employing only logical terms. It is for this reason that it is necessary to find a definition of 'number' in logical terms, a definition which Russell accomplished by use of a logical construction of 'number'.

The analysis of numbers, descriptions, and classes can be treated as analyses of particular concepts disregarding metaphysical considerations. But the analysis, to Russell, was carried out assuming a definite metaphysics and philosophy of logic. His logical system is not a system independent of metaphysics, as can be seen from an examination of *Principles of Mathematics*. Thus the logical analysis of these concepts which Russell provides will at the same time be an analysis of the metaphysical nature of numbers, descriptions, and classes.

The general philosophical position which Russell supported at this time can be called an extreme realist position, extreme when compared to other positions which have been called realist, and extreme when compared to Russell's own later philosophy. He gives most credit for his views to G. E. Moore, and they can best be summed up by the following quotations:

> On fundamental questions of philosophy, my position, in all its chief features, is derived from Mr. G. E. Moore. I have accepted from him the non-existential nature of propositions (except such as happen to assert existence) and their independence of any knowing mind; also the pluralism which regards the world, both that of existents and that of entities, as composed of an infinite number of mutually independent entities, with relations which are ultimate, and not reducible to adjectives of their terms or of the whole which these compose. . . . The doctrines just mentioned are, in my opinion, quite indispensable to any even tolerably satisfactory philosophy of mathematics, as I hope the following pages will show (xviii).[1]

[1] Throughout this Part, references in the text by page numbers only are to the *Principles of Mathematics*.

That every presentation and every belief must have an object other than itself and, except in certain cases where mental existents happen to be concerned, extra-mental; that what is commonly called perception has as its object an existential proposition, into which enters as a constituent that whose existence is concerned, and not the idea of this existent; that truth and falsehood apply not to beliefs, but to their objects; and that the object of a thought, even when this object does not exist, has a Being which is in no way dependent upon its being an object of thought: all these are theses which, though generally rejected, can nevertheless be supported by arguments which deserve at least a refutation.[1]

Among the entities which must have Being Russell includes propositions. Frequently the distinction is made between 'sentences', written or spoken series of words, 'propositions', what these sentences 'mean', and 'facts', or states of affairs, which are the actual events which the propositions purport to be about. Russell accepts the existence of sentences, which express propositions, but his notion of 'proposition' curiously combines propositions in the more usual sense of the word with states of affairs. 'A proposition, we may say, is anything that is true or that is false' (12–13). Propositions are not only entities in their own right, but the physical objects or concepts mentioned by means of the proposition are themselves actually constitutents of it. If an individual man or material object is the subject of a proposition, that individual or object is itself a constituent of the proposition; the proposition is a unified whole composed of such objects and whatever other entities are found to enter into it. Propositions will not then 'correspond' with states of affairs, since those affairs themselves are among its constituents.

This at once raises the problem: What is a fact? And the difficulty of this problem lies in this, that a fact appears to be merely a true proposition, so that what seemed a significant assertion becomes a tautology.[2]

[1] Russell, 'Meinong's Theory of Complexes and Assumptions', *Mind*, N. S. XIII (1904), 204. Russell says that Frege and Meinong are epistemologists nearest this position.

[2] *Ibid.*, p. 523.

This conception of propositions holds regardless of whether they are true or false. Russell is unable to distinguish between the metaphysical status of true and false propositions, and consequently maintains that false propositions also have Being.[1]

Judgment is an attitude toward propositions; we may believe, or not believe, a true or false proposition. '. . . there are, apart from and independently of judgment, true and false propositions, and . . . either kind may be assumed, believed or disbelieved.'[2] But the theory of propositions which Russell has adopted makes it difficult to analyse the notion of truth. The difference between a true and false proposition appears to be one that we directly apprehend.

> It may be said—and this is, I believe, the correct view—that there is no problem at all in truth and falsehood; that some propositions are true and some false, just as some roses are red and some white; that belief is a certain attitude towards propositions, which is called knowledge when they are true, error when they are false.[3]

The difficulty with the view can be seen in the case of a true and false proposition concerning the same individual, who is a constituent of both propositions. Both propositions will be composed of equally real constituents, such as the individual in question and various concepts or relations, and both will have the same metaphysical status, yet one proposition is true and one false, a difference that apparently can only be immediately apprehended. It is difficulties in clarifying this notion of truth which led Russell himself to abandon it shortly after he had first published it.[4]

[1] Russell, 'Meinong's Theory of Complexes and Assumptions', *Mind*, N. S. XIII (1904), pp. 510–11.

[2] *Ibid.*, p. 522.

[3] *Ibid.*, p. 523.

[4] See Russell, 'The Nature of Truth', *Mind*, N. S. XV (1906); for his earlier view see *Principles of Mathematics*, pp. 3, 35, 48, 504, and *Mind*, 1904, 522 ff.

It is interesting to compare this general view of the relation of propositions and sentences with that of the later 'logical atomism' period, see below, Chapter III, Part II. In this later period, 'proposition' has approximately

The objects of all our thoughts and perceptions have Being, and Russell introduces a general word, 'term', to apply to any of these objects:

> Whatever may be an object of thought, or may occur in any true or false proposition, or can be counted as *one*, I call a *term*. This, then, is the widest word in the philosophical vocabulary. I shall use as synonymous with it the words unit, individual, and entity. The first two emphasize the fact that every term is *one*, while the third is derived from the fact that every term has being, *i.e. is* in some sense. A man, a moment, a number, a class, a relation, a chimæra, or anything else that can be mentioned, is sure to be a term; and to deny that such and such a thing is a term must always be false (43; cf. also 449).

Whether or not something is a term can always be ascertained by discovering if it can be made the subject of a true or false proposition, in short, whether it can occur as a substantive.[1] Further, terms are immutable, indestructible, identical with themselves and diverse from all other terms. 'Numerical identity and diversity are the source of unity and plurality; and thus the admission of many terms destroys monism' (44). It should be noted that material objects are among the entities which Russell finds to be terms. At this time Russell believed that we perceive material objects directly; there is no mention of a mental content, or sense-datum, mediating between the perceiver and the external physical object. It is essentially a theory of a naive realism, although it contains complexities that I shall not consider here.[2]

The relation between sentences and propositions is that a sentence 'means' a proposition in the sense that the words in a sentence indicate or denote constituents in the corresponding proposition. Russell does not speak of the meaning of an entire

the same meaning as 'sentence' has here, and the objective reference of 'propositions' is to 'facts'. A 'fact', however is close to what Russell means by 'proposition' in this earlier period.

[1] It is true, though not clear from the preceding quotation, that not every constituent of a proposition is necessarily a 'term'; only those constituents are terms which can be made subjects of further propositions.

[2] *Mind*, 1904, 215–17.

sentence, since he does not find one object that the sentence as a whole denotes;[1] rather the sentence must be analysed into its constituent parts, and these parts will denote corresponding constituents of the proposition that the sentence 'means'. Where the subjects of propositions are physical objects, the subject word in the corresponding sentence will be a 'proper name' and it will denote or 'mean' the particular physical object in question, which will be a constituent of the proposition (44). For general words, such as verbs and adjectives, the case is not so simple, but usually a general word will denote a concept that will be a constituent of the corresponding proposition (52, 502).

The material that logic investigates are the relations between propositions, propositions themselves, and the nature of their constituents, such as classes and relations. This material all has an objective existence in the world apart from any of the activities of logic or philosophy. Logic is primarily a matter of formulating the perceived relations between propositions, becoming able to discern whatever basic entities are essential to the system, thus making explicit the logical relationships and logical constants present in the world. Deducing pure mathematics from logic thus provides mathematics with a firm foundation since it becomes based on an objective logical structure. The nature of philosophical analysis for Russell at this time is consistent with this conception of logic. Propositions are objective entities, composed of real constituents, and analysis of propositions is *real* analysis since it will reveal these real constituents. There is no such thing, Russell says, as 'conceptual' analysis, where what is meant is the analysis by the mind of what is objectively indivisible. Rather all analysis is *real*, since 'where the mind can distinguish elements, there must *be* different elements to distinguish; though, alas! there are often different elements which the mind does not distinguish' (466;

[1] This view can be contrasted with that of Frege, to whom a sentence denotes a 'truth-value'. See Russell's criticism of Frege, *Principles of Mathematics*, p. 503.

cf. also 129). The only caution in analysis is that while analysing a proposition will reveal its constituents, it must be remembered that the proposition cannot be simply reconstituted by placing its constituents side by side. It is a whole which is not simply the sum of its parts.[1]

The analysis of 'number', of 'descriptions', and of 'classes', will then attempt to discover what terms, if any, are constituents of propositions expressed by sentences containing symbols apparently denoting these entities. In the case of numbers, it is an attempt to discover whether symbols for numbers denote some entity 'number', and if not, what terms, if any, such symbols do denote. I have mentioned that it is important for Russell that he find some analysis of 'number', as in fact he does, that will show symbols for numbers do not denote some special kind of entity 'number', but rather entities that have already been introduced into the logical system. Russell's conception of logic has made his problem of the definition of number not merely a symbolic problem, but at the same time one of metaphysics, of finding those entities that are really involved in the notion of 'number'.

The analysis of number which Russell undertakes is not that of 'number' in its most general meaning, but of the different kinds of numbers required by mathematics. The first step is the definition of 'cardinal number'. The series of finite cardinal numbers, or 'natural numbers', 1, 2, 3, 4, . . . *n* . . are the usual numbers of daily life and elementary arithmetic. This series of cardinals can easily be obtained once the term 'cardinal number' has been defined by means of the logical system which Russell develops; further relations between cardinals, such as the familiar arithmetical operations of addition and multiplication, can then be defined by means of principles already contained in the logical system.[2]

[1] *Principles of Mathematics*, pp. 49, 466. For a fuller discussion of Russell's later meaning of 'analysis', see below, Chapter V, Part I.

[2] In Russell's works, the definition of number which I shall present is given first in *Principles of Mathematics*, Part II. The technical symbolic

THE CONSTRUCTION OF NUMBERS,

Numbers seem in common use to be associated with classes, to be, in fact, properties of classes. Thus when we speak of a dozen eggs, or of $22 in someone's wallet, we mean that the class of eggs contained in the box has a dozen members, or that the class of dollars in the wallet has 22 members. Whether or not there are other uses of numbers, this would seem to be the essential one. It is not always obvious that when we speak of the number of something we are speaking of the number of members of a class, for we do not always make explicit our reference to the class; but attention to the process of assigning numbers will, I believe, always reveal some class whose members we are referring to. We do not give numbers to objects at random, even enumerating some miscellaneous group of objects will form the class 'objects I have just enumerated'. The number of a group of objects is, then, the number of members of a class of which all the objects in question are members. Noting the connection between classes and numbers is the first step in formulating a definition for 'number'.

Ordinarily, we say that the number of objects in two or more groups are 'equal' if there is the same number of objects in each of the groups. The simple method of determining equality is to count the objects in the groups, and see if they have the same number. It is possible, however, to give a meaning to 'equal' which will not involve use of the term 'same number' of objects.

formulation is found in *Principia Mathematica*, Vol. II, Part III. Russell gives a clear, simplified account in the *Introduction to Mathematical Philosophy*, Chapters I–IV, and in *Our Knowledge of the External World*, 1st edition, pp. 200 ff. Ernest Nagel's essay in *The Philosophy of Bertrand Russell* gives an excellent brief survey of Russell's definition, with a critical discussion of Russell's claim of 'eliminating' entities, pp. 319 ff.

Many of the notions which Russell uses, especially that of 'equivalence' or 'similarity' between classes, or sets, will be found in books on the mathematical theory of sets, or aggregates, e.g. Fraenkel, *Mengenlehre*, Chapter I; Courant and Robbins, *What is Mathematics?* Chapter II. Among works which develop the basic concepts of the theory of sets and also employ Russell's definition of number are J. W. Young, *Lectures on Fundamental Concepts of Algebra and Geometry*, which gives a precise, clear exposition, and two more general works, A. Dresden, *Invitation to Mathematics*, and Kasner and Newman, *Mathematics and the Imagination*.

This is done by making use of the technical term 'similarity' or 'equivalence' between classes (or 'sets' or 'aggregates' in mathematics). 'Similarity' is defined by means of 'one-to-one correspondence' between objects. A familiar illustration of one-to-one correspondence is an infantry company where rifles are issued to each soldier; each soldier will have a rifle, assigned by the serial number on the gun, and only as many rifles will be issued as there are men in the company. It is unnecessary, in order to establish this correspondence, that the number, either of the soldiers or of the guns, be known. It could be established by having the soldiers in line file past the gun racks, each man taking one, and only one, rifle. When all the soldiers have filed past the gun racks, there will be a one-to-one correspondence between the men and the rifles they have taken, and the number of men and guns will be 'equal'. Although we do not know what the number of each is, the class of men and the class of rifles will be 'similar', they will have the same 'number'. More technically, two classes are similar if they can be placed into a one-to-one correspondence whereby some relation pairs every element of one class with a certain element of the other class, and vice versa.

At this point it is important to examine the definition of similarity to see if it presupposes knowledge of the meaning of the cardinal number '1'; if so, its use will be considerably curtailed as a means of defining number in general since the particular number '1' will have been assumed undefined. Russell maintains that 'similarity' does not presuppose an analysis of the cardinal number '1', although on this point many of his critics disagree; they maintain that his definition of number is circular, since it involves knowledge of the meaning of '1' in establishing a one-to-one correspondence. Russell discusses this point at some length (113, 132, 135), the result is that he finds it necessary to distinguish between 'a term' and '1'. 'A term' is indefinable, and refers chiefly to what can be the logical subject of a proposition; while '1', as we shall see, is capable of definition. The objects which are placed into a

one-to-one correspondence are not counted; it is sufficient merely to discriminate them from each other, and discriminating between objects does not necessarily imply that we place them in classes, and predicate numbers of those classes. Russell gives a definition of a one-to-one relation without using '1': 'R is a one-one relation if, when x and x' have the relation R to y, and x has the relation R to y and y', then x and x' are identical, and so are y and y'' (130).

If the relation of similarity can be established between two classes, we say that they have the 'same number'. Similarity is a reflexive, symmetrical, transitive, relation; it has the familiar properties of the relation of equality, or of that of identity: a class is similar to itself; if a class u is similar to a class v, then v is similar to u; and if u is similar to v, v similar to w, then u is similar to w. A relation such as similarity, or identity, in fact, all reflexive, symmetrical and transitive relations, may be thought of as based upon some common property that the various terms possess. Thus 'number' may become (as Russell says it did for Peano) the common property that is held by similar classes. This is a plausible view, for similar classes seem to have one thing if nothing else in common, their number. Six eggs, six people at a party, six horses running on a track, are alike only in being classes of six objects. They are similar in virtue of having the property of being six. Such a conception of number may be used as a definition 'by abstraction', starting with similar classes we 'abstract' from the nature of the individual elements and from the order in which these elements are arranged, leaving only the number of the objects. In such a way we may arrive at Cantor's definition:

> We will call by the name 'power' or 'cardinal number' of M the general concept which, by means of our active faculty of thought, arises from the aggregate M when we make abstraction of the nature of its various elements m and of the order in which they are given.[1]

[1] This definition was originally published by Cantor in *Mathematische Annalen*, Vol. 46 (1895), p. 481. The quotation is from the translation by

Russell does not accept such a definition by abstraction; it fails to define *the* number of a class, since it cannot be proved by the method of abstraction that there is not an infinite number of entities which could be the required common property.

Instead of trying to locate the common property of similar classes, let us consider the class of those similar classes. The members of that class will be classes similar to each other, no members can be found that are not similar to all the others, and the members will all have the 'same number' of terms. Let us call this class of classes the 'number' of any of the similar classes which are its members. Then 'cardinal number' is defined as 'the class of all classes similar to the given class'.[1] Two illustrations of this definition are the definitions of '0' and '1'[2] which are not only important numbers in themselves, but which also illustrate the method by which any definite number can be defined. '0' is the class of classes whose only member is the null class; '1' is the class of all classes which are not null and are such that, if x and y belong to the class, then x and y are identical (128; 132).[3]

It is important to see clearly what it is Russell has defined, since otherwise we may expect the definition to have too wide an applicability. The definition is of 'cardinal number' in general, the notion of *any* cardinal, not the series of cardinals. The properties of the set of cardinal numbers may be obtained starting from Russell's definition of any cardinal, as I shall explain below. It is the properties of the series of numbers that are important for mathematical purposes (241), and which are given in such systems of axioms as Peano developed.

P. E. B. Jourdain, *Contributions to the Founding of the Theory of Transfinite Numbers*, La Salle, Illinois, 1941, p. 86. See another quotation from Cantor on p. 74 of Jourdain.

[1] *Principles of Mathematics*, p. 115; see also *Principia Mathematica*, II, *100.

[2] *Principia Mathematica*, I, *52.

[3] This definition of number had also been arrived at by Frege independently a few years before Russell discovered it. Russell learned of his work only in time to discuss it in an Appendix to the *Principles of Mathematics*. *Principles of Mathematics*, App. A, pp. 519 ff.

Peano employs three primitive ideas, o, finite integer (number), successor of, and in addition five axioms.[1] This system of primitive ideas and axioms is sufficient to give the properties of the series of cardinal numbers; after further definitions, operations and more complex relations between numbers, e.g. addition and multiplication, can be derived. Such a system clearly defines the serial properties of the number system, the fact that numbers form a progression. For Russell's purpose, however, a system such as Peano's has one fatal disadvantage, as there is no unique set of terms defined by the axioms which will correspond with what we ordinarily consider the series of natural numbers. The only way in which this correspondence can be established with Peano's system is to simply assume that his three primitive notions are what we commonly mean by those words. Unless we assume that meaning they are capable (as Russell shows in *Introduction to Mathematical Philosophy*, Chapter I) of an infinite number of interpretations; and thus what Peano's system actually defines is any progression of the form $x_0, x_1, x_2, \ldots x_n, \ldots$ in which there is a first term, every term has a successor, there is no last term, no repetitions, and every term can be reached in a finite number of steps.

It is not difficult, given Russell's definition of 'cardinal number', to define the class of finite numbers, which will be one definite class that will satisfy Peano's axioms. We have seen how Russell defines 'cardinal number', and 'o' and '1'; 'successor of' may then be defined approximately in this way: if n be a number, then $n+1$ is a number formed by adding a unit to the class of n terms. Finite numbers will then be those that belong to every class to which o belongs, and to which $n+1$ belongs if n belongs. (This definition incorporates the principle of mathematical induction.) (128) The advantage of proceeding from his definition of number, Russell feels, is that we know now that there is at least one definite class, that of the natural numbers, which satisfies the axioms of arith-

[1] See Russell's account of Peano in *Principles of Mathematics*, Chapter XIV, *Introduction to Mathematical Philosophy*, Chapters I, III.

metic, and furthermore gives to these numbers the same meaning as they have in daily life.

The definition of cardinal number which I have outlined is one of Russell's best known accomplishments, and represented a necessary step in his derivation of mathematics from logic without the introduction of any peculiarly mathematical notions. On the basis of his definition of number, he finds it simple enough to introduce the usual arithmetic operations. There are, however, other types of numbers besides cardinal numbers and the question might arise whether they would have to be introduced as undefined entities. Russell proceeds to show that those, too, can be defined in terms of notions already in his logical system, and need not be considered special types of entities any more than it is necessary to consider cardinal numbers such entities. Fractions, ratios, positive and negative numbers, he finds can be introduced not as new types of numbers, but as relations between cardinal numbers. The reader can find a discussion of these relations in *Principles of Mathematics*, Chapter XVIII, and *Introduction to Mathematical Philosophy*, Chapter VII. Real numbers, however, are numbers which cannot be so introduced, and are numbers for which some definition must be found; Russell's procedure in defining them is again an example of what later he called 'logical constructions'. I shall present his definition at some length, following some remarks on his definition of cardinal numbers.

The importance of Russell's definition of 'cardinal number' is that numbers no longer remain indefinable entities which must be 'apprehended' (127). It is this definition which enables mathematics to be deduced from logic, for number is defined in terms of logical constants, by means primarily of the notion of class. Russell speaks often of having 'eliminated' numbers; the preceding discussion has made clear, I believe, that what has been eliminated is not numbers in every sense of the word, but a special kind of real entity denoted by numerals.[1] There

[1] Contrast this view with *Principles of Mathematics*, p. 43, above p. 23, where numbers (distinct from classes) are included as one kind of term. I

might, however, in addition to the classes which define numbers by some common property, such as that given by the definition by abstraction, which could be an apprehended entity 'number'; Russell cannot see any reason for dogmatically denying the existence of such a common property, but he feels that the assumption of its existence is unnecessary, for

> Wherever Mathematics derives a common property from a reflexive, symmetrical, and transitive relation, all mathematical purposes of the supposed common property are completely served when it is replaced by the class of terms having the given relation to a given term; and this is precisely the case presented by cardinal numbers (116).

In the case of numbers, and in the constructions which he was later to make, the result is that he does not prove that certain entities do not exist, but he finds that there is no evidence that they do exist, and since the function which they served can be equally served by a construction, there is no useful purpose served in assuming that they exist.

The quotation just given appears to state a general methodological principle which Russell's definition of cardinal numbers exemplifies, and which we could expect to see exemplified in all cases where it is applicable. His later formulation of the use of logical constructions as a methodological procedure would confirm such an impression.[1] In Russell's definition of cardinal numbers it has been seen that he rejected their definition 'by abstraction'. However, elsewhere in *Principles of Mathematics* he accepts the 'Principle of Abstraction' and employs it in formulating certain definitions. It is a principle which Russell adopted from Peano; it asserts that in any given symmetrical, transitive, reflexive relation, e.g. that of similarity or equality, the terms of the relation all have a further relation to some other term, which can be called their

regard this statement as a slip, since the preceding discussion implies that numbers are no longer one kind of term, but terms only insofar as classes are terms.

[1] See below, Chapter V, Part II.

'common property'. The principle corresponds to the general statement that things are equal because they have some property in common. More precisely stated, the Principle of Abstraction is as follows:

> . . . whenever a relation, of which there are instances, has the two properties of being symmetrical and transitive, then the relation in question is not primitive, but is analyzable into sameness of relation to some other term; and that this common relation is such that there is only one term at most to which a given term can be so related, though many terms may be so related to a given term (166; cf. also 220).

This statement asserting the existence of one term to which all the equal terms are related should be compared with the earlier discussion of the principle in connection with the definition of cardinal number,[1] where the existence of this *one* common entity is explicitly denied (114) since Russell maintains that an infinite number of such common entities could exist. The reader may well be puzzled whether we should assume that Russell maintains the Principle of Abstraction should be followed or should be dispensed with.

The advocacy of the Principle of Abstraction, as stated in the preceding quotation, is in connection with Russell's analysis of equal quantities and magnitudes. The principle is used as a support for the theory of absolute position. In discussing equal magnitudes, Russell finds that a magnitude, such as one of length, is a simple indefinable concept, and cannot be equal to another magnitude. 'Equality' belongs to quantities, and it is only between quantities that we can speak of the relation of equality holding. Equality consists in possession of the same magnitude, since following the Principle of Abstraction, a class of equal quantities, e.g. measurements of length, are equal because they have the same magnitude. On the relational theory of position, equal distances have no further quality than that of being members of a class of equal quantities; but on Russell's absolute theory equal quantities belong to a class of

[1] See above, pp. 29, 32.

equal quantities all of whose members have one property in common, that of having the same magnitude. However, in spite of this supposition of the one common entity, Russell goes on to say that a magnitude may, as far as *formal* purposes are concerned, be identified with the class of equal quantities (167n), a procedure similar to that used in the definition of cardinal number.

Whatever Russell believes to be the status of the Principle of Abstraction, as compared to his dispensing with abstractions in his definitions of numbers in the *Principles of Mathematics*, in his later works it is a principle which he definitely abandons in favour of the opposite procedure. Again, in later definitions, as in that of number, he does not maintain that he has proved that the common entity, or common entities, supposed by the Principle of Abstraction do not exist, but that we have no reason for assuming that they do. If their existence is found to be substantiated, or at least desirable, on other grounds, then we can accept that existence. Apparently, in his theory of magnitude, it was grounds, aside from the formal analysis, connected with the general theory of absolute position which led him to invoke the Principle of Abstraction.

In many mathematical text-books numbers other than the positive integers 1, 2, 3, . . . are introduced as 'extensions of the number concept', their introduction justified by their use in solving certain geometrical or arithmetical problems. Once the need becomes apparent, and the success of the new numbers in filling that need is clear, no further justification for their existence is thought necessary. Thus the operations of addition and multiplication can be carried out without restriction among the positive integers, but subtraction is only possible when the number subtracted is smaller than that from which it is subtracted. It is made possible always to carry out subtraction by the introduction of negative numbers, e.g. $4 - 7 = -3$. Even in this more inclusive class of both the positive and negative integers, division could not be carried out at all times, for the quotient of two integers is not necessarily an integer,

.g. 7/4. The introduction of fractions which, together with the ositive and negative integers, form the class of rational umbers, enables the four arithmetic operations to be uni-ersally carried out, so that the equation $ax=b$ always has a olution (assuming that o is not involved).

Practical and geometrical considerations also were served y the introduction of these new numbers. Fractions made ossible the measurement of lengths (the usual case, in fact) vhich cannot be expressed as an integral multiple of the unit ength, e.g. $7\frac{3}{4}$ feet. It is the characteristic of rational numbers o form a series which has an infinite number of members etween two whole numbers, and is such that between any wo rational numbers there is always a third. When these num-ers are represented by points on a line, the line will apparently e completely covered by points, since between any two points, o matter how close together, there will always be a third. It vould seem that all geometric lengths would correspond to ome point on this line, and thus could be measured by some ational number. However, it was soon found that even all he rational numbers were not sufficient, or that the points n the line corresponding to these rational numbers did not xhaust all the points on that line. Thus, e.g. the diagonal of square whose sides are 1 unit long is $\sqrt{2}$ units long, but the oint terminating a segment of this length does not correspond o a rational number, that is, the equation $x^2=2$ has no ational number for a solution. We can of course approximate o the value of $\sqrt{2}$ by series of rational numbers, enclosing he $\sqrt{2}$ in an ever-narrowing gap by finding a series of numbers vhose squares are all less than 2, and also by a series whose quares are all greater than 2. We are able to calculate values or numbers like $\sqrt{2}$, 'irrational numbers', and, since they are seful, assume them to exist; irrational, together with rational umbers, then compose the series of real numbers.

The procedure just described does not really, however, stablish the existence of these numbers other than cardinal umbers, nor does it make clear their nature. Why does such

an equation as $x^2 - 2 = 0$ necessarily have a solution? Why is it not equally justifiable to devise a new kind of number for every equation which up to the present has no solution? The fact that there are geometrical lengths corresponding to rational and irrational numbers has no relevance to those numbers as part of arithmetic, but only to numbers as applied to geometrical measurements. The procedure appears to introduce new numbers merely by postulating them whenever there is a need which they will satisfy, certainly a convenient process. 'The method of "postulating" what we want has many advantages; they are the same as the advantages of theft over honest toil. Let us leave them to others and proceed with our honest toil.'[1] The new numbers, Russell maintains, should be defined in arithmetic terms, in terms of the number system itself, as having certain definite arithmetic relations and properties.

In the past, the definition of irrationals was commonly effected by geometrical considerations. This procedure was, however, highly illogical; for if the application of numbers to space is to yield anything but tautologies, the numbers applied must be independently defined; and if none but a geometrical definition were possible, there would be, properly speaking, no such arithmetical entities as the definition pretended to define (278).

Or again, it was argued that the diagonal of unit square has evidently a precise and definite length x, and that this length is such that $x^2 - 2 = 0$. But such arguments were powerless to show that x is truly a number. They might equally well be regarded as showing the inadequacy of numbers to Algebra and Geometry (280).

I have mentioned that once having defined cardinal number Russell is able to define positive, negative, and rational numbers. The attempt to give a rigorous definition of irrational and real numbers had been made by various mathematicians, of whom Dedekind was one of the most successful. I shall briefly sketch his method preliminary to giving Russell's definition of real and irrational numbers.[2]

[1] *Introduction to Mathematical Philosophy*, p. 71.

[2] I shall not discuss the definition of complex numbers, which may be found in *Principles of Mathematics*, Chapter XLIV.

The extension of the number system by the abitrary means I have described fails to distinguish clearly between the properties of the different kinds of numbers introduced. Thus the series of cardinal numbers is 'extended' by adding to it the series of negative integers, and this combined system further extended to become the series of rational numbers. The series of rational numbers, in turn, by the addition of the irrational numbers, becomes the series of real numbers. The logical properties of each series changes, however, as each extension is made. The important properties of the series of cardinal numbers is that it is a discrete series which has a first term; on the other hand, the series composed of both positive and negative integers, although a discrete series, is now one that has no first term. When this series in turn is extended to become that of the rational numbers, it is no longer even a discrete series, but a compact one, a series with a term between any two terms, no matter how close together. Each extension of the system of numbers has brought with it a change in the logical character of the system, the discrete series of integers when combined with fractions has produced the compact series of rational numbers.

The definitions of numbers which Russell gives makes clear the difference between the various series of numbers. Positive and negative integers are not the same kind of numbers as cardinal numbers, the latter are classes of classes, while the former are relations between cardinal numbers. The series formed of positive and negative numbers is a different series from that of the cardinal numbers, and does not include the latter as a sub-class. The cardinal numbers should not be identified with the positive whole numbers; the series 1, 2, 3, . . . is not the *same* series as +1, +2, +3, . . ., but rather has a one-to-one correspondence with it. Similarly, the fractions 1/1, 2/1, 3/1, . . . chosen from the series of rational numbers, *correspond*, but are not the same, as 1, 2, 3, . . . An analogous result will be found to hold for real and rational numbers. The introduction of new numbers does not then simply extend the old

system, but requires the formulation of a new system, part of which will have a one-to-one correspondence with the older system.[1]

One of the methods frequently used in mathematics to 'define' irrational and real numbers is that of Dedekind, who employed the notion of 'cuts'. The set of rational numbers has the characteristic of forming a compact series, i.e. a series such that between any two members there is always a third member, no matter how close together the first two are. Such a series is distinguished from a discrete series, e.g. that of the natural numbers, where there are two members, say 2 and 3, between which no other member lies. The set of rational numbers can be divided in many ways into two classes, which may be called A and B, such that every member b of B is after (i.e. is larger) than every member a of A. A division of the rational number

[1] The 'extension of the number concept' can be followed in many books on mathematics, including those given below. Especially clear is F. Waismann, *Einführung in das Mathematische Denken*, Vienna, 1936, Chapters I, II, who also gives the criticism I have presented.

An exposition of Dedekind's 'cuts' will be found in many mathematical books on the foundations of mathematics or on the theory of functions of real variables, e.g. J. W. Young, op. cit., and Courant and Robbins, op. cit. Dedekind himself presented it in *Stetigkeit und irrationale Zahlen* (1872), English translation by Beman in *Essays on the Theory of Numbers* (Dedekind), Chicago, 1924.

Russell's discussion of real and irrational numbers, and his criticism of Dedekind, is in *Principles of Mathematics*, Chapters XXXIII, XXXIV, *Introduction to Mathematical Philosophy*, Chapter VII. I find *Introduction to Mathematical Philosophy* the most intelligible discussion.

It is interesting to note in Courant and Robbins, op. cit., pp. 68 ff., an example of what Russell calls a 'construction'. In deriving irrational numbers by nested intervals on the number axis, they say one may *define* 'an irrational point as just a *symbol for a certain sequence of nested rational intervals*. From this aspect an irrational point *is* simply a sequence of nested rational intervals with lengths tending to zero. Hence our fundamental postulate really amounts to a definition. To make this definition after having been led to a sequence of nested rational intervals by an intuitive feeling that the irrational point "exists", is to throw away the intuitive crutch with which our reasoning proceeded and to realize that all the *mathematical properties* of irrational points may be expressed as properties of nested sequences of rational intervals' (69).

series in this way Dedekind calls a 'cut'. There are three possibilities in making cuts, of which only one can hold in any particular instance:

1. There is a largest element of A, but no smallest element of B, e.g. A contains all rational numbers less than or equal to 2, B contains all rational numbers greater than 2, and 2 is largest element of A.

2. There is a smallest element of B, but no largest element of A; e.g. B contains all (rational) numbers greater than or equal to 2, A contains all numbers less than 2, and 2 is the smallest element of B.

3. There is neither a largest nor a smallest element in either A or B; e.g. A contains all numbers less than the $\sqrt{2}$, and B contains all numbers greater than the $\sqrt{2}$.

Cases (1) and (2) cannot both hold; for supposing that a were the largest element of A, and b the smallest element of B, there would be a rational number $a+b/2$ which would lie between a and b and not be in either class A or B, thus contradicting the hypothesis that we can divide the rational numbers into two classes such that there is a largest element of one class and a smallest element of the other.

In cases (1) and (2) there is a rational number which 'effects' the separation of the set of rational numbers into two classes, or, roughly speaking, there is some rational number which marks the point of division into two classes. It is either the largest element of one class or the smallest of the other; in the examples given it was the number 2. In case (3), however, what is it that determines the cut? It would seem reasonable to suppose that there is something which produces the cut, since the cut is perfectly definite, consisting, e.g. of all numbers whose squares are greater or less than 2. Similar cuts can also be formed corresponding to all other irrational numbers, the result being two infinite series of rational numbers converging toward the place where the irrational number is presumed to lie. It cannot, however, be a rational number that produces these cuts; for in that case it would belong to one

of the two classes and would be either the greatest or smallest element of one of them. It is also not necessary that there should be a rational number between the two classes of case (3); for as Russell shows (280), although between any two rational numbers there is always a third, yet this result does not hold for any two *classes* of rationals; the two classes may exhaust all the rational numbers leaving none in between.

Dedekind said that in the case where the cut was not produced by a rational number, that 'we create a new, an *irrational* number a, which we regard as completely defined by this cut (A_1, A_2); we shall say that the number a corresponds to this cut, or that it produces this cut'.[1] The system of numbers that corresponds to all cuts is the real number system; this system contains both rational and irrational numbers, distinguished by the kind of cut corresponding to them. Irrational numbers have not, like rational numbers, been introduced and characterized prior to making cuts, but have been introduced as useful in giving every cut a number that produces it.

Irrational numbers might also be characterized as the 'limit' of converging series of rational numbers, i.e. the upper and lower parts of an irrational cut might be said to approach a limit, which will be the irrational number that in the preceding paragraph was said to produce the cut. Russell's discussion of the nature of 'limit' indicates his belief in the dubious status of irrational numbers introduced in Dedekind's fashion. In Russell's discussion of limit, the limit of a class of terms forming a series is found to be not itself a member of that class, but instead is a member of a larger class within which the former series is included. 'A limit may be defined generally as a term which immediately follows (or precedes) some class of terms belonging to an infinite series, without immediately following (or preceding, as the case may be) any one term of the series' (277). If the limit of a class of terms is itself a member of that class, it could not immediately follow any other term of the

[1] *Essays on the Theory of Numbers*, p. 15.

class, since between any two terms of an infinite series there is always a third term. However, if the limit is not a member of the class of terms of which it is the limit, then it is possible for it to be the term which immediately follows all members of the class. In rational cuts, the set of rational numbers is divided into two parts between which lies the one rational number that produces the cut. This 'limit' of the two parts of the cut can still be a rational number, since both it and the two classes of numbers are contained in the set of rational numbers. However for irrational cuts, the two parts of the cut exhaust the set of rational numbers; if we are to assume a limit which the two parts of the cut approach it cannot be a rational number. Following Dedekind's assumptions, the limits of these irrational cuts, or irrational numbers, will be included in a larger system of numbers, the real numbers, that also includes the rational numbers.

The introduction of irrationals by Dedekind's method has the disadvantage of making real numbers simply the combined set of rational and irrational numbers. Can we establish the relations of greater or less than, or the ordinary arithmetical operations, within a class containing such diverse entities as rational and irrational numbers? Irrational numbers are not, like rational numbers, relations between pairs of integers. The definition of greater and less than that holds for rational numbers must be redefined to hold between irrationals, or between rationals and irrationals. Russell formulates such a definition (283), but it requires as a consequence the formation of a new series that includes irrational numbers and terms having a one-to-one correspondence with rational numbers. Thus in determining the nature of irrational numbers, not only is it irrational numbers which are in question, but the nature of the system of real numbers.

Although Dedekind's results are a definite improvement over the introduction of irrational numbers on the basis of geometrical or algebraic requirements, they do not *define* irrationals, but merely *specify* them.

> Granting the existence of irrationals, they are merely specified, not defined, by the series of rational numbers whose limits they are. Unless they are independently postulated, the series in question cannot be known to have a limit; and a knowledge of the irrational number which is a limit is presupposed in the proof that it is a limit . . . from rational numbers alone, no proof can be obtained that there are irrational numbers at all, and their existence must be proved from a new and independent postulate (282).

Irrational numbers introduced in Dedekind's fashion are an example of 'postulated' entities, or of what Russell later called 'unknown' entities. The existence of such entities is an inference apparently justified on the basis of certain definite needs, or interrelationships between entities previously introduced, in this case the rational numbers. Russell's later formulated principle of logical constructions maintained that such unknown entities should, if possible, be replaced by 'logical constructions'. The definition which Russell gives of real number and irrational number is an example of such a construction, although at the time he made the definition he did not speak of it as an example of a more general procedure. His definition of real and irrational numbers gives them the required arithmetical properties without requiring the existence of a postulated entity; further, the relations within the system of real numbers can be expressed without the difficulties I discussed in the preceding paragraph caused by the difference between rational and irrational numbers.

We can understand Russell's definition of real number by returning to the notion of Dedekind cuts. Dedekind showed that the series of rational numbers could be divided into two classes A and B, such that every member of A is smaller than every member of B. Let us consider only the lower part of this cut, the class A, and call this a 'segment'. Then segments have the properties, (1) if a is an element of the segment, any element which precedes a is also an element of the segment; (2) the segment has no last element. (A segment could alternatively be defined as the upper part of the cut instead of the lower, or as the whole cut, both upper and lower parts. The following discussion would

hold whichever method is chosen.) As we have seen, any rational number can define a cut, or similarly define a segment, e.g. all numbers less than or equal to 2 is a segment. A 'terminated' segment is defined as a segment with an upper limit. According to Russell's definition of limit, the limit of a class of terms, such as a segment, is not itself a member of that class, but the first term following the class. The segment, then, even though having a limit, will have no last term. Every rational number 'defines', or corresponds to, one and only one segment which will be a terminated segment. The segments defined by rational numbers do not, however, exhaust all the possible segments, any more than the rational cuts exhaust all the possible cuts; for we still have remaining those segments that have no limit, that are not terminated. Even though there are segments that are not terminated, it is still possible to arrange all segments in serial order. Of any two different segments one must be part of the other, i.e. one segment must include terms which are not in the other; the relation of greater than among numbers will correspond to the logical relation of inclusion among segments. We can order the segments without any reference to the rational numbers with which they correspond. Arranging all segments of rationals in serial order, Russell then *defines* a segment to be a real number. Those segments that are terminated are defined to be rational real numbers, those segments that are not terminated are defined to be irrational real numbers. The series of rational real numbers corresponds one-to-one with the series of rational numbers, defined as pairs of integers, but they are not the *same* numbers. Those real numbers which are not rational are irrational; they are segments which are not terminated, e.g. $\sqrt{2}$ becomes the segment consisting of all rational numbers whose squares are less than 2.

The definitions of cardinal and real number I have presented were formulated to make possible the derivation of mathematics from logic. This was done by showing that in expressions containing symbols for numbers it was not necessary to assume that

the symbols denoted special kinds of entities, but rather that the expressions could be replaced by others employing, instead of these symbols, only logical terms previously introduced. Without these definitions symbols for cardinal and real numbers would be assumed to denote an entity, 'cardinal number' or 'real number', which at the least would be assumed or postulated, and whose further metaphysical status would be a matter of investigation. If we grant the adequacy of Russell's definitions, not only can we determine that pure mathematics can be derived from logic, but also that it is not necessary to assume the existence of numbers as special kinds of entities.

It does, of course, now remain to determine whether Russell's definition is adequate. I feel, for the most part, that it is. Such a simple statement on my part is not sufficient, but it would be difficult to give an adequate examination of Russell's results without an extended technical discussion. In the absence of such a discussion, I shall refer the reader to various works which have questioned Russell's definition.[1] There are sufficient favourable accounts, I feel, to make references to them unnecessary.

In the first place, it is clear that his definition has made possible the derivation of pure mathematics from logic, as the system of *Principia Mathematica* demonstrates. The fact that there are certain problems raised by this system does not mean that the definition of number was responsible for them, or that other definitions of number would remove them. Arguments questioning the system of *Principia Mathematica* are not, I feel, arguments necessarily questioning his definition of number.

Not only is Russell's definition adequate for logical purposes, as I shall assume that it is, but he also maintains that it is

[1] Several criticisms of Russell's definition are presented and answered by J. Jorgensen, *A Treatise of Formal Logic*, Copenhagen and London, 1931. Criticisms depending on fundamental differences as to the nature of mathematics can be found in such writers as Poincare, Helmholtz, Cassirer. See also Waismann, op. cit., pp. 85 ff.

Reichenbach, in *The Philosophy of Bertrand Russell*, also undertakes to reply to certain criticisms of Russell's definition, pp. 31 ff.

adequate as an analysis of what 'number' means in daily life. His construction, then, serves to relate the abstract system of arithmetic to the objects to which that system is applied in practical calculations. The series of cardinal numbers, without an interpretation of 'number', is an abstract series concerning certain entities in certain relations; to make this series useful in counting actual objects it is necessary to interpret the entities of the series so that they have essentially the same meaning as the numbers of daily life. Russell maintains his definition is such an interpretation.[1]

In determining whether Russell's definition accords with what is generally 'meant' by numbers, it would be simpler to limit this question to determining whether his definition agrees with the way in which numbers are *used*. What is commonly meant by numbers may include not only beliefs as to their uses, but metaphysical views, explicit or otherwise, concerning the status of numbers. I shall disregard such possible metaphysical associations. The illustrations I gave when presenting Russell's definition make clear that his definition of number will account for their usual practical functions. Some of the works cited above, however, question this fact, but the considerations they advance I do not find compelling enough to discuss. I am not aware that Russell made similar claims about his definition of real numbers. It seems less likely that real numbers as he defines them do conform to common usage, but on the other hand their 'common usage' is largely limited to a mathematical one. If one maintains his definition of real numbers is not adequate, it would be necessary to show that real numbers as Russell defines them, even though serving all mathematical purposes of real numbers, would not exhaust what the mathematician 'means' by such numbers.

The importance which one attaches to Russell's claims that he has analysed the common 'meaning' of number depends on the place it is felt such an enterprise occupies in his work. His

[1] *Our Knowledge of the External World*, 1st edition, p. 204; *Introduction to Mathematical Philosophy*, pp. 9, 18.

main task is apparently that of analysing number to make possible the derivation of mathematics from logic, and in so doing to eliminate the need for assuming numbers as entities. These purposes he has achieved. Is it necessary that an analysis fulfilling these aims should also be such as to give the common meaning of number? It would be convenient if it did, so that the same definition could be used in the formulation of the system of pure mathematics, and at the same time as an empirical interpretation of it. But it is possible that the definition could serve Russell's purposes and still not be an adequate empirical interpretation. In this case, besides Russell's definition, we would require an additional interpretation to make the system of pure mathematics applicable to daily life. If then, Russell's principal aims are those I have mentioned, as I believe they are, it is not an objection to the fulfilment of those aims that his definition is not what we 'mean' by number. Thus even if one should deny (as I do not) that his definition of cardinal number, and even more of real number, conforms with common usage, the value of his definition would still remain.

PART II

DESCRIPTIONS

The analysis of a sentence, Russell maintains, is accomplished by determining the constituents of the proposition that it expresses, whether they be existing objects or concepts. This process will make clear the meaning of the sentence in question. The analysis of statements containing symbols for numbers is an example of such an analysis, and Russell maintains his conclusions show what is really meant by such statements, what are the actual constituents that are referred to. Certain types of sentences offer considerable difficulty, however, especially those containing descriptive phrases. Russell's analysis of these sentences resulted in his well-known 'theory of descriptions', which has been widely accepted by contemporary

logicians, whether or not they agree with Russell's general position.

Sentences containing descriptions may be contrasted with those containing 'proper names'. The notion of 'proper name' is not an easy one to comprehend precisely, but at present it is sufficient to accept as a proper name names of persons, places or things in the usual sense. The function of these names is to refer to some definite object or place, and their meaning is primarily that object or place to which reference is made. Thus 'H. E. Jones', if there were such a person, would serve to designate that person, and its meaning would be that person. 'New York' is a name designating a city, and its meaning is that city.

The analysis of sentences (or 'propositions' in Russell's later terminology of the logical-atomism period) containing names offers comparatively little difficulty. Thus in 'I met H. E. Jones' the 'I' denotes the person making the statement, 'H. E. Jones' designates the person met. Both persons are constituents of the proposition expressed by the sentence. (Or the 'fact' expressed by the proposition, according to Russell's later view.) In 'New York is a large city', 'New York' denotes the actual city New York, a constituent of the proposition. The significance of this kind of statement lies in the relation that is asserted to hold between the two persons, or the properties that the object or person is asserted to possess.

'Descriptions', however, are phrases of the form 'a so-and-so' or 'the so-and-so', whose function can be seen in such statements as 'I met a man', or 'the largest city in the United States is New York'. Russell distinguishes between descriptions which are indefinite, or ambiguous, and those which are definite, which concern only one object. The examples which I have given are, respectively, an indefinite and a definite description. 'Description' is used in the familiar sense of the word where we refer to an object of a kind specified by giving some of the distinguishing characteristics of that kind of object, or to one particular object singled out by means of the properties which

that object possesses. It is not always simple to determine whether what appears to be a proper name is really a name or is a description. 'New York' is frequently used as a name, but may be used as a description, in which usage it is an abbreviation for a longer description beginning 'the city which . . .' and giving all the characteristics, or at least the most essential, which one has associated with that city. Similarly, 'Homer', although it looks like a name, is a description in terms of such characteristics as 'author of the *Odyssey* and *Iliad*' or 'being blind'. Descriptions occur in every subject-matter, for example, in mathematics 'the integer between 2 and 4' describes the number 'named' by '3'. This example shows that not only can objects be directly and unambiguously referred to by names, but they can be unambiguously, even though indirectly, referred to by a description in terms of their properties and relations to other objects. This indirect reference can be as definite as naming, so that an object can be unambiguously spoken of without a sign being given especially to it. Not every object must have a sign all its own; it can be referred to by means of general meanings and signs already given to other objects to which it is in some way related.[1]

At first sight it might seem that sentences containing descriptive phrases could be analysed in the same manner as those containing names, since, after all, the descriptive phrase is used to refer to some one, or some definite, object. To take one of Russell's examples, 'Scott is the author of *Waverley*', since 'the author of *Waverley*' presumable refers to Scott, it might be thought to mean Scott in the same way as does the name 'Scott'. But if this were the case, if 'the author of *Waverley*' means Scott in the same sense as a proper name, why not replace the phrase with the shorter name 'Scott' which would serve the same function more concisely? Now what we have is 'Scott is

[1] In addition to the kind of descriptions of which I have given examples above, there are also what Russell calls 'descriptive functions', e.g. 'the father of x', or 'the square root of 3'. Descriptive functions, though important, need not be further considered, since their analysis offers no further difficulties in principle.

Scott', which is certainly true, but hardly what was meant by the original statement. A similar result follows if we assume that an indefinite descriptive phrase can be analysed as if it were a proper name. If we say 'I met a man', and if 'a man' is assumed to refer to the particular man we did in fact meet, say Jones, then we could replace the original statement by 'I met Jones'. But this is clearly not the meaning of the original statement.

This analysis fails, first of all, because it does not account for the significance of descriptive phrases. A description refers to an object by means of the characteristics or properties which that object possesses; in fact, there may be no object satisfying the description at all, but it is still significant to speak of an instance of certain properties. We must find some way to analyse descriptions so that their reference to objects through characteristics and properties, their distinguishing difference from proper names, can be explained.[1]

Further, this analysis runs into serious difficulties if the object

[1] It might be thought that such an explanation is given by the distinction which Frege gives in his paper 'Über Sinn und Bedeutung'. He distinguishes between the 'meaning' and 'indication' or 'denotation' of a sign. It is then significant to assert identity between an object named and one described because in the first case the 'denotation' of the sign is used, while in the second, the 'meaning' (or 'intension', or 'connotation') is used. Such an assertion of identity is possible because the object referred to is the same, and it is significant because the terms employed have different 'intensions' (or none at all, if one term is employed as a name, in its denotative aspect). A similar distinction was used as a theory of descriptions by Miss E. E. Constance Jones, who engaged in a running controversy with Russell for several years on descriptions. I do not find either of these theories as adequate as Russell's, and consequently am not giving them any further exposition. The reader can find them in the following sources:

G. Frege, 'Über Sinn und Bedeutung', *Zeitschrift für Philosophie und Philosophisces Kritik, C* (1892), 25-50 (reprinted, in translation, in Feigl and Sellars, *Readings in Philosophical Analysis*).

Russell's criticisms of Frege's distinction as applied to descriptions can be found in his 1905 paper, 'On Denoting' (cited below).

E. E. C. Jones, papers in *Mind*, N. S. XIX (1910), XX (1911), and in *Proc. Arist. Soc.*, N. S. XI (1911).

Russell's criticisms of Miss Jones are given in his 1911 paper (cited below).

described is not an existent object. If the object described does exist, the assumption that the descriptive phrase, like a proper name, denotes the object described raises no difficulty with the metaphysical status of that object, even though the assumption robs the descriptive phrase of its peculiar significance. But if the object described does not exist, either as a matter of fact, for example, 'the 50th State of the United States', 'a unicorn', or cannot exist, for example, 'the round-square', then we must determine the metaphysical status of these objects. It might be maintained that non-existent, but possible, objects 'subsist', as also might be the case with scientific and logical entities. Can it be maintained that self-contradictory objects subsist? If so we seem to be faced with terms having Being whose nature is self-contradictory.

The analysis of descriptions which Russell gave in the *Principles of Mathematics* was an attempt to explain the significance of descriptive phrases, as this previous analysis I have given failed to do, and yet assume the Being of the object described as a 'term'. About this same time, the Austrian psychologist and philosopher Meinong offered a different analysis which, like Russell's, tried to explain the significance of descriptions and at the same time keep the object described as an entity of some kind. I shall discuss each of these analyses briefly, and I believe the discussion will indicate the difficulties in the assumption that the object described is a term of some sort. Russell himself soon after his first theory abandoned the belief that the object described was a term, and advanced a new theory (1905) which in a more precise formulation was incorporated into his logical system in *Principia Mathematica*.[1]

[1] For Russell's account of descriptions in the *Principles of Mathematics*, see Chapter V of that work. The first formulation of his later theory is given in 'On Denoting', *Mind*, N. S. XIV (1905), 479–93 (reprinted in Feigl and Sellars, *Readings in Philosophical Analysis*). Russell's most complete and technical account of his theory is in *Principia Mathematica*, I, 66 ff., and I, *14. Another account of his later theory is given in 'Knowledge by Acquaintance and Knowledge by Description', *Proc. Arist. Soc.*, N. S. XI (1911), 108–28 (reprinted in *Mysticism and Logic*, New York, 1918). The reader may find

In the *Principles of Mathematics* Russell believed that sentences containing descriptive phrases could be analysed so that the constituent of the proposition corresponding to the descriptive phrase would be what he called a 'denoting concept'.

> A concept *denotes* when, if it occurs in a proposition, the proposition is not *about* the concept, but about a term connected in a certain peculiar way with the concept. If I say 'I met a man', the proposition is not about *a man*: this is a concept which does not walk the streets, but lives in the shadowy limbo of the logic-books. What I met was a thing, not a concept, an actual man with a tailor and a bank-account or a public-house and a drunken wife.[1]
>
> When a class-concept, preceded by one of the six words, *all*, *every*, *any*, *a*, *some*, *the*, occurs in a proposition, the proposition is, as a rule, not *about* the concept formed of the two words together, but about an object quite different from this, in general not a concept at all, but a term or complex of terms. This may be seen by the fact that propositions in which such concepts occur are in general false concerning the concepts themselves . . . In such cases, we say that the concept in question *denotes*.[2]

A denoting concept is a kind of concept expressed by a phrase formed by a word indicating a class-concept preceded by one of the six words listed in the quotation. It is a denoting concept that is a constituent of propositions expressed by such sentences as 'I met a man'. The object denoted is of a different kind in each of the six cases, either a term or a complex of terms; although this object is not a constituent of the proposition containing the denoting concept, that proposition is yet 'about' the object denoted.

the most lucid, non-technical account to be that of *Introduction to Mathematical Philosophy*, Chapter XVI. Many modern works on mathematical logic also carry accounts of Russell's later theory.

In *The Philosophy of Bertrand Russell*, there is a critical, though generally favourable, detailed account of Russell's theory by G. E. Moore, 'Russell's "Theory of Descriptions" ', pp. 175–225. Briefer accounts in that book are by M. Weitz, pp. 92 ff., Max Black, pp. 240 ff., and a criticism by K. Gödel, pp. 128 ff.

[1] *Principles of Mathematics*, p. 53.

[2] *Ibid.*, p. 64.

Whether or not there is an important difference between the six different kinds of denoting concepts, Russell does not discuss; he does, however, believe that the relation of denoting is the same in all cases, and the important difference lies in the nature of the objects denoted. I shall assume that one can distinguish between the six kinds of denoting, as illustrated by all men, every man, any man, a man, some man, the man. Russell gives an elaborate discussion of this distinction, but he is not certain of the nature of the object denoted in each case. 'All men', at least, seems to denote the class of men. The other cases are less clear, for example,

> *Some man* must not be regarded as actually denoting Smith and actually denoting Brown, and so on: the whole procession of human beings throughout the ages is always relevant to every proposition in which *some man* occurs, and what is denoted is essentially not each separate man, but a kind of combination of all men. This is more evident in the case of *every*, *any* and *a*.[1]

Russell concludes this much:

> There is, then, a definite something, different in each of the five cases, [*the* is discussed separately] which must, in a sense, be an object, but is characterized as a set of terms combined in a certain way, which something is denoted by *all men*, *every man*, *any man*, *a man*, or *some man*; and it is with this very paradoxical object that propositions are concerned in which the corresponding concept is used as denoting.[2]

The use of denoting concepts explains, Russell believes, the significance of descriptions.

> If we say 'Edward VII is the King', we assert an identity; the reason why this assertion is worth making is, that in the one case the actual term occurs, while in the other a denoting concept takes its place. . . . Often two denoting concepts occur, and the term itself is not mentioned, as in the proposition 'the present Pope is the last survivor of his generation'. When a term is given, the assertion of its identity with itself, though true, is perfectly futile, and is never made outside the logic-books; but where denoting con-

[1] *Principles of Mathematics*, p. 62.
[2] *Ibid.*

cepts are introduced, identity is at once seen to be significant. In this case, of course, there is involved, though not asserted, a relation of the denoting concept to the term, or of the two denoting concepts to each other.[1]

If this theory is to be accepted as an adequate theory of descriptions it will be necessary to clarify the notion of 'denoting concept'. A 'denoting concept' is similar to ordinary concepts in that it is certain abstract properties or characteristics, but with the further property of referring to objects having those properties. As Russell says, it can 'inherently and logically *denote* such terms'.[2] Granted these properties, it may be seen that the use of denoting concepts explains the significance of descriptions, but it is not at all clear what kind of peculiar concept it is that can have these properties. Russell's later theory can be viewed as a development and clarification of this notion of denoting concept, for that theory showed that descriptive phrases refer to concepts or properties, and that the object described is an instance of these concepts.

The theory also requires that objects described, or 'denoted' by denoting concepts, whether existent, non-existent, or self-contradictory, be terms and have Being, even though they are not constituents of propositions containing the denoting concept.[3] The fact that the denoted objects are non-existent or self-contradictory makes them no less terms. But, as in the case of numbers, Russell came to believe that it was desirable, if at all possible, to dispense with attributing Being when it could be avoided, and especially with impossible objects. This aim, he maintained, he had accomplished in his later theory of descriptions.

The epistemological distinctions which Meinong developed can also be used as an explanation of descriptions.[4] He main-

[1] *Principles of Mathematics*, p. 64.

[2] *Ibid.*, p. 53.

[3] That objects described were terms can also be seen from Russell's discussion of his general philosophical position at this time. See above, p. 23.

[4] A. Meinong, 'Über Gegenstandstheorie', in *Gesammelte Abhandlungen*, Vol. II, Leipzig, 1913; *Untersuchungen zur Gegenstandstheorie und Psychologie*,

tained that every idea which we have of an object corresponds to an object which either (1) exists, or (2) subsists, such as 'ideal' objects like numbers and relations, or (3) can be considered as a 'pure' object, indeterminate both as to being and non-being, for example, the 'round-square', or the 'golden mountain'. It is possible, Meinong believes, to make statements about objects of the last type which will be true, for example, 'the round-square is round'; since these objects are considered apart from the characteristic of having Being, and apart from normal logical connections and relations, we avoid the contradiction met when the 'round-square' is assumed to subsist. Not only do our ideas of objects in every case correspond to a particular object, but judgments and statements correspond to what Meinong calls 'objectives'. These are analogous to what Russell at this time called 'propositions'. True statements refer to objectives which subsist; false statements refer to objectives which, like pure objects, possess various definite determinations, but cannot be thought of as having either Being or non-Being. This complex set of distinctions was presumed to make it possible to maintain that every idea and statement corresponds to an object without at the same time requiring that false statements and self-contradictory notions should refer to objects which have Being, as did Russell's 'terms' and false propositions.

If Meinong's theory as so far developed were applied to the analysis of descriptions, the object described would in every case be an entity, but whether or not that object had Being would depend on further considerations. It would then be in accord with Russell's early theory in assuming that the object described is always an entity, but would avoid the difficulties Russell encountered of attributing Being to impossible objects. It is still necessary, however, to see why descriptions are sig-

Leipzig, 1904. An English account of Meinong which discusses all the points relevant to the present discussion is J. N. Findlay, *Meinong's Theory of Objects*, London, 1933. Russell discusses Meinong in several papers appearing in *Mind*, N. S. XIII (1904), N. S. XIV (1905), and N. S. XVI (1907).

nificant. Meinong introduces further distinctions which are designed to answer this question.[1]

Besides the objects which so far Meinong has discussed, we find further that there are 'incomplete' objects. An incomplete object differs from a complete object in being determined only in regard to a certain finite number of properties, while a complete object is determined either as to the possession or to the lack of possession of every possible property. Existent or subsistent objects are complete objects, for they must be determinate in every respect; incomplete objects which are not merely pure objects cannot then have being in either of these two senses. Whenever we describe an object, for example, 'a man' or 'the man', an incomplete object is used to refer, and to lead us to comprehend, some complete object. The exact relationship between incomplete objects and the complete objects they lead us to is obscure; Meinong says they have a kind of being called 'implexive' and that they are 'embedded' (or *implektiert*) in the complete object.

An object can be referred to either by pointing to the object itself and giving it a name, or by means of one of the incomplete objects embedded in it. Thus 'the man' refers to any object which includes among many other incomplete objects that one which is determined only by the characteristic of being human. The assertion of identity between an object named and the same object described is significant as an analysis of the object showing that it possesses a certain property, that is, that a certain incomplete object is embedded in it. We think of the object described not as the incomplete object it in fact is, but as the complete object that can never be fully known, in which it is embedded. This is possible by stipulating explicitly that our reference is not to something incomplete, but to a complete object, for determinateness and indeterminateness are not part of the nature of the object, and can be added without producing any change in its nature. We now have, finally,

[1] A. Meinong, *Über Möglichkeit und Wahrscheinlichkeit*, Leipzig, 1915. J. N. Findlay, op. cit.

three kinds of objects: (1) complete objects, (2) incomplete objects which are used to refer to the complete objects, and (3) a modified form of (2), in which determinateness is explicitly added to incomplete objects.

The principal difficulty with Meinong's theory to my mind is that it is unbelievable. It also appears to me to be an example in epistemology of what Russell referred to in another connection as the method of 'postulation'. Wherever different types of reference are encountered, the difficulty is resolved by Meinong through the introduction of another kind of entity which has the properties just required to solve the problem at issue, and no unnecessary properties which might raise further difficulties. There are few problems which could not be solved by such a method as this, provided one requires no further evidence for the existence of such entities than their function in solving problems. Russell rejected Meinong's analysis primarily because he came to believe that as few entities as possible should be assumed; Meinong's world with its many kinds of objects was to him a completely 'unreal' world.[1]

The later analysis of descriptions which Russell carried out avoids the 'unreal' entities that Meinong assumed, and that Russell himself had at first assumed. At the same time Russell is able to explain the significance of descriptions by finding that statements containing descriptive phrases express propositions concerning an instance of various propositional functions, which represent the 'properties' that those (or the) instances (if any) possess. In order to avoid assuming that the object described is itself a constituent of the proposition, Russell analyses, not descriptive phrases 'in isolation', but the entire sentence in which they occur. Descriptive phrases become, then, 'incomplete symbols'. Instead of defining only 'a so-and-so' or 'the so-and-so', he defines rather 'the (or 'a') so-and-so has the property ϕ' or 'the (or 'a') so-and-so exists'. If we were to assume that descriptive phrases had meaning in isolation, we

[1] *Introduction to Mathematical Philosophy*, p. 169: Russell, 'My Mental Development', in *The Philosophy of Bertrand Russell*, pp. 13–14.

would, Russell maintains, be confronted with the situation met in the other theories I have discussed of assuming some object which such phrases would denote. The phrases would mean some object, either an existent one, or the peculiar object corresponding to 'a man', or the impossible one corresponding to 'the round-square'. This problem does not arise when descriptive phrases are treated as incomplete symbols.[1] The analysis which I shall present now is one, then, which treats descriptions as incomplete symbols and does not require that the object described be a constituent of propositions corresponding to descriptive phrases. It is an additional example of what Russell later called a 'logical construction'.

In sentences containing indefinite descriptive phrases, 'a so-and-so', Russell's analysis shows that the statement concerns a propositional function, $\phi\hat{x}$, which is satisfied by several (one or more) values of x.[2] In a statement about 'a man', the property 'being human' will be represented by ϕ, and any assertions about 'a man' will be about the several values of x having the property ϕ. The correct analysis of propositions predicating some property of 'a so-and-so' is that there are one or more objects x, having the property ϕ, which makes it 'a so-and-so', and the further property ψ. Thus, 'the joint assertion of ϕx and ψx is not always false',[3] or, '$(\exists x).(\phi x.\psi x)$'. Where further properties are predicated of 'a so-and-so', as in the preceding example, the resulting proposition is not of the form 'ψx'. If this were the case, then 'a so-and-so' would be an argument for the function ψ, and 'a so-and-so' would be identical with 'x', and hence a descriptive phrase would mean an object. It is precisely this result which Russell wishes to avoid. The difference between his analysis of the assertion of further properties of 'a so-and-so', that is '$(\exists x).(\phi x.\psi x)$' and '$(\exists z).(\psi z)$', where z represents ϕx, is evident. The former reads

[1] For a further discussion of incomplete symbols, see below, Chapter V, Part II.

[2] *Principia Mathematica*, I, 30.

[3] *Introduction to Mathematical Philosophy*, p.171.

'there is at least one object which has the properties ϕ and ψ'; and the latter, 'there is at least one so-and-so that has the property ψ'. The former statement has eliminated 'a so-and-so' and is concerned only with the instances of the joint occurrence of two properties.

Definite descriptions are analysed in a manner similar to indefinite ones by replacing the expression containing the descriptive phrase with one that asserts that some object x exists, or that x has one or more properties; but in this case it must also be stated that there can be no more than one object having the designated combination of properties. A definite description, then, implies uniqueness; we cannot say truly '*the* inhabitant of New York', because there is no such unique object, the phrase is an indefinite description satisfied by several million values of x. On the other hand, 'the author of *Waverley*' is a description that can be satisfied by only one entity, and we know that Scott is that one entity. Often it may happen that what we thought was an indefinite description turns out to be definite; and that what seemed to be a definite description is really indefinite, as would be the case if we found that *Waverley* was really written by two men. If, however, we have asserted that the description in question is definite and it later appears that it is really indefinite, then our original proposition was false. Russell maintains that a proposition asserting a definite description is false unless there is actually no more than one object to which it applies. Symbolically, the difference between the two kinds of descriptions can be seen by eliminating the condition expressing uniqueness and the result will be the form expressing an indefinite description, '$(\exists x).(\phi x.\psi x)$'.

In any definite description, such as Russell's example 'the author of *Waverley* was a poet', three statements are involved which must be expressed in its symbolic formulation:[1]

(1) There was an author of *Waverley*, i.e. *Waverley* was written, and presumably written by some person;

[1] *Introduction to Mathematical Philosophy*, p. 177; *Principia Mathematica*, I, 68.

(2) No more than one person wrote *Waverley*;

(3) That one person was a poet.

None of these three conjunctive factors can be omitted because the falsity of any one of them makes the entire proposition false; there is no intermediate ground beside the truth or falsity of the whole proposition. Thus, 'the author of *Waverley* was a poet' is false if:

(1) No one wrote *Waverley*; or

(2) *Waverley* were written by more than one person; or

(3) Whoever wrote *Waverley* was not a poet.

A proposition concerning descriptions is false whether it (1) asserts various properties of a non-existent object, or (2) asserts that there is only one object, when there are really several such objects, or (3) is correct in asserting that there is one and only one object having certain properties, but false in asserting a further property of this object. Common usage seems also to distinguish these three cases; using the previous example, 'the authors of *Waverley* were poets' is false according to (2), while 'the author of *Waverley* was a playwright' is false by (3). 'The authors of *Waverley* were playwrights' is false according to both (2) and (3); a proposition that Scott was the author of some non-existent book, or of a book that someone else wrote, is false by (1); and a proposition asserting that some non-existent individual wrote some non-existent book is also false by (1). What might be called the different kinds of falsity of a descriptive statement is not considered by Russell's theory, the statement is false without regard to which of the alternatives is false. This, however, is not a serious disadvantage; if one wanted to distinguish between descriptions according to the reason for their falsity a specific statement could be added to this effect.

Symbolically, the three parts involved in a statement containing a definite description are expressed in this way: the first, '$(\exists x).\phi x$', which means that there is at least one object satisfying $\phi\hat{x}$; and the second, '$\phi x.\phi y.\supset_{x,y}.x=y$', which means that only one object satisfies it, that is, that any two values

of $\phi\hat{x}$ are identical, so that there can be no more than one such value. The two expressions taken together can be formulated as '$(\exists c): \phi x. \equiv_x . x = c$'. (Using the previous example, 'ϕ' is 'wrote *Waverley*'.) This says that there is a term c, such that 'x is the author of *Waverley*' is true when and only when x is identical with c. The statement is an equivalence according to the definition of equivalence, since ϕx is true only when x is identical with c, and is false when x is not c; the two parts of the expression are then consequently either both true or both false, and so are equivalent. We have, in the expression given here, found a means of symbolically formulating a proposition which observes conditions (1) and (2); for the expression states that at least one person wrote *Waverley*, and that no more than one wrote it. Russell introduces the expression as the definition of 'the x satisfying $\phi\hat{x}$ exists'; thereby defining not the descriptive phrase in isolation, but a statement within which it occurs.

The third condition can now be met by adding to the expression already developed a further function of c, such as fc, or any number of such functions.[1] This is a function of c, rather than of x, because it is a property of the definite individual c, which has the property ϕ, the only value for which $\phi\hat{x}$ is true. The resulting proposition is written '$(\exists c): \phi x. \equiv_x . x = c : fc$', and this will be found to fulfil all the three required conditions. It will be false if c does not have the property f, because both of the conjuncts in the proposition must be true if it is to be true as a whole; or the proposition will be false if the first conjunct is false, which will occur under the conditions described in the preceding paragraph. In Russell's logical system the proposition which is read 'the x satisfying ϕx satisfying fx', or, for example, 'the individual who wrote *Waverley* was also a poet', is defined as '$(\exists c): \phi x. \equiv_x . x = c : fc$'. Thus the descriptive phrase is defined as an incomplete symbol, the entire sentence in which it occurs is defined, rather than the descriptive phrase by itself.

[1] *Introduction to Mathematical Philosophy*, p. 178; *Principia Mathematica*, I, 68.

Any proposition of the form exemplified by 'Scott is the author of *Waverley*' may be analysed using the method just developed.[1] It is analysed as '$(\exists c) : \phi x . \equiv_x . x = c : c = a$' which means 'there is a c for which $\phi x . \equiv_x . x = c$, and this c is a', in this case, 'there is a c for which it is true that "the author of *Waverley* is c" and this c is Scott'. The complete expression is equivalent to $\phi x . \equiv_x . x = a$, because $x = c$ and $c = a$, and $x = a$; since a is a definite known object no variable and quantifier c is needed. The expression is read, using the preceding example, ' "x wrote *Waverley*" is always equivalent to "x is Scott" '; since 'x wrote *Waverley*' is true when and only when 'x is Scott'. It is in this way that propositions are analysed which state that an object described is the same as one having a definite name, preserving the significance of the proposition, and at the same time explaining how the object named and the object described are identical.[2]

This analysis of descriptions is successful in explaining the significance of descriptions, and at the same time avoiding making the object described a constituent of the proposition in question. It is, I believe, an adequate analysis of descriptions since it clarifies the meaning of descriptions as commonly used. However, its importance for Russell's later work, like that of the analysis of numbers, is that it shows that descriptive phrases can be treated as incomplete symbols; statements in which these phrases occur can be replaced by equivalent statements not containing such phrases. It is not then necessary to assume,

[1] *Principia Mathematica*, I, 69.

[2] In the preceding discussion I have been concerned only with propositions which are entirely composed of the assertion of the existence of an object described or of predicating some property of that object. In such propositions a description is said to have a 'primary' occurrence. There is, however, a more general case to be considered in which the descriptive statement is only a part of the proposition. The simplest example might be ' "the author of *Waverley* was a poet" is false'. In such cases a description is said to have a 'secondary' occurrence. I am not considering the secondary occurrence of descriptions here, since they offer no new difficulties in principle. A detailed analysis of secondary occurrences of descriptions can be found in *Introduction to Mathematical Philosophy* and *Principia Mathematica*.

merely because of the occurrence of a descriptive phrase, that some object corresponding to that phrase must exist. It is an additional illustration that by means of an appropriate analysis it is possible to dispense with certain entities formerly assumed to exist.

The theory of descriptions also turned out to have an epistemological function for Russell. It is the epistemological analysis that it helped to provide which will be met with in increasing frequency in Russell's later writings. Russell distinguishes between 'knowledge by acquaintance' and 'knowledge by description'; it will be seen that the theory of descriptions can be applied in relating the two kinds of knowledge. This distinction I shall present here largely as Russell himself first extensively presented it.[1] The distinction is one which I shall not criticize at this point, for its significance can be seen better after a consideration of Russell's later writings. It is an early attempt at formulating an aim which has great importance in Russell's later philosophy, that of determining in what way and to what extent empirical knowledge is ultimately based on the individual's sensible experience. The theory of descriptions Russell believed showed how knowledge by description could be related to the individual's own experience.

When we know an object by acquaintance, Russell says, it is directly presented to us, when we know it by description it itself need not be present, but instead a reference to it by means of several of its characteristics. 'I shall say that an object is "known by description" when we know that it is "*the* so-and-so", i.e. when we know that there is one object, and no more, having a certain property; and it will generally be implied that we do not have knowledge of the same object by acquaintance.'[2] A large part of our knowledge is of this kind; for example, we say 'the father of x', or 'the product of 14 and 13',

[1] 'Knowledge by Acquaintance and Knowledge by Description', *Proc. Arist. Soc.*, N. S. XI (1911), 108–28 (reprinted in *Mysticism and Logic*, New York, 1918); *The Problems of Philosophy*, London and New York, 1912, Chapter V.

[2] *Proc. Arist. Soc.*, 1911, 113.

where we do not know the individual or number directly by its being present before us, but know only that there is a unique object of such-and-such a kind. It is more difficult to understand what precisely constitutes knowledge by acquaintance; Russell states: it is '. . . a direct cognitive relation to that object, i.e. when I am directly aware of the object itself. When I speak of a cognitive relation here, I do not mean the sort of relation which constitutes judgment, but the sort which constitutes presentation.'[1] 'We shall say that we have *acquaintance* with anything of which we are directly aware, without the intermediary of any process of inference or any knowledge of truths.'[2] It might seem as if the most obvious example of knowledge by acquaintance was knowledge of physical objects, but Russell holds that this is not the case; for both physical objects and other persons' minds are known only by description.[3] In *The Problems of Philosophy* he states that we are acquainted with such particulars as sense-data, facts of memory, facts of introspection, the self, and with universals such as concepts of qualities and relations. But the notion of acquaintance is a difficult one, and one to which I shall devote some attention in the following Chapter.

The importance of knowledge by description is that it allows us to go beyond our own private experience, to learn what others have experienced, and what we can never hope to. But we can never by using descriptions involving only universals obtain knowledge about any particular beyond that which follows logically from those descriptions.[4] For example, 'the most long-lived of men' applies only to one man, but we know no more about him than that he will be the longest-lived of

[1] *Proc. Arist. Soc.*, 1911, p. 108.

[2] *The Problems of Philosophy*, p. 73.

[3] *Ibid.*, p. 81. It is sometimes convenient to use examples which imply that physical objects and other persons' minds can be known by acquaintance; but this is in contexts, or preliminary discussions, where the precise distinction would make the discussion unnecessarily complex. The reader may have noted that Russell also at times does this, but this is purely for the sake of simplicity and does not imply any fundamental change in his views.

[4] *Ibid.*, p. 87.

men. If we are to know that our descriptions apply to the empirical world, they must refer in some way to a particular with which we are acquainted. Any description known to be truly applicable to a particular must, Russell maintains, 'involve some reference to a particular with which we are acquainted. . . .'[1] This is one sense in which Russell's statements can be interpreted that 'all our knowledge, both knowledge of things and knowledge of truths, rests upon acquaintance as its foundation';[2] and that '. . . knowledge concerning what is known by description is ultimately reducible to knowledge concerning what is known by acquaintance.'[3] Any statement referring to an existent object can eventually be reduced to one which contains the actual sense-data connected with the particular object, or the 'self' of the person referred to, as a constituent.

The preceding quotations are, however, also interpreted by Russell in a more general sense which leads to more significant consequences. All knowledge, Russell maintains, should be reduced to knowledge by acquaintance in the sense that every proposition which we can understand must be composed of constituents with which we are acquainted, and that every expression should be put in such a form that it enables us to see that the proposition it expresses is one that is composed of constituents known by acquaintance. I shall give several similar quotations on this point: 'The fundamental epistemological principle in the analysis of propositions containing descriptions is this: *Every proposition which we can understand must be composed wholly of constituents with which we are acquainted.*'[4] 'All propositions intelligible to us, whether or not they primarily concern things only known to us by description, are composed

[1] *The Problems of Philosophy*, p. 87.

[2] *Ibid.*, p. 75.

[3] *Ibid.*, p. 91.

[4] *Ibid.*, p. 91 (italics in original); *Proc. Arist. Soc.*, 1911, 117. Max Black, in *The Philosophy of Bertrand Russell*, pp. 244 ff., discusses this principle at some length. His criticisms are answered by Russell in the same volume, pp. 691 ff.

wholly of constituents with which we are acquainted, for a constituent with which we are not acquainted is unintelligible to us.'[1]

The last sentence of this quotation indicates why the principle is required, that is, a constituent of a proposition with which we are not acquainted is unintelligible to us. If, following Russell, we are completely to understand a statement, it must first be analysed revealing the constituents of the proposition to which it corresponds. It is then necessary for these constituents to be intelligible to us and, according to Russell, this is possible only when we are acquainted with them. The application of this principle offers no difficulty as far as universals (qualities and relations) are concerned, since Russell includes them among the kinds of things with which we have acquaintance. It does, however, require a theory of descriptions to make clear what it is we are acquainted with in the case of propositions corresponding to statements containing descriptive phrases. The analysis of these statements is especially important since Russell maintains that physical objects and other persons can only be known by description.

If Meinong's analysis of descriptions were accepted, the constituent of a proposition corresponding to a descriptive phrase would be an object, either existent, or subsistent, or a 'pure' object. In understanding such propositions it is one of these types of objects with which we would be acquainted. It is Russell's later theory, however, which provides an analysis of descriptions that makes it unnecessary to assume that we are acquainted with objects described. A statement containing a descriptive phrase, according to this analysis, is equivalent to a statement that refers to a proposition containing concepts and the notion of an instance of certain concepts. There is no difficulty with being acquainted with these constituents, and there is no question of having to be acquainted with the object described since that object is not a constituent of the proposition. In statements asserting the identity of an object named

[1] *Proc. Arist. Soc.*, 1911, 128.

and an object described, we can be acquainted with the object named, since proper names at this time Russell considered were only applicable to things with which we were acquainted, and the other constituents of the proposition are those mentioned above.

Since physical objects are known by description, propositions can refer to them by means of descriptions without necessitating any actual experience of the objects. If, however, these propositions are to be shown to refer to an individual's sense experience, some reference to actual sense-data must be made. This can be accomplished, for example, by stating that such-and-such sense-data belong to the object in question. These sense-data are known by the individual by acquaintance, and consequently propositions concerning physical objects can be reduced to propositions containing constituents with which the individual is acquainted. Such an analysis will also fulfil the requirement that any description known to be truly applicable to a particular must 'involve some reference to a particular with which we are acquainted. . . .' It will be seen that Russell's later construction of physical objects also accomplished this aim.

PART III

CLASSES

In examining numbers and descriptions, Russell has succeeded in formulating an analysis which makes it unnecessary to assume the existence of entities corresponding to symbols for numbers and descriptions. The notion of 'class' is also of fundamental importance for logic and its analysis similarly raises the question whether there is an entity 'class' corresponding to class symbols. Here, as in the previous cases, Russell was able to show that statements concerning classes could be analysed without the necessity for the assumption of an entity 'class'. I shall barely more than outline Russell's theory, since for my purposes it is only an additional illustration of the use

of logical constructions, and further because an adequate discussion would involve highly technical considerations which would be aside from the main point with which I am concerned.

We can begin with the notion of 'class' in its ordinary usage, where it refers to all objects which are alike in some one definite respect, which we can say possess a common property or characteristic. For example, we might consider the class of men, the class of mice, the class of Republicans, the class of men who won the war. The problem then is to decide if 'the men who won the war' or 'mice' as used in the preceding examples denote kinds of entities called 'classes', or, if not, how propositions corresponding to statements containing these phrases can be analysed. It is the latter alternative which Russell adopts to arrive at a final solution, but he had at first adopted the view that classes are entities.

When the *Principles of Mathematics* was written, Russell assumed that classes were entities, an assumption which he abandoned shortly afterwards when he found that a more adequate analysis by means of logical constructions could be given in which 'class' becomes an incomplete symbol. This final version can be found in its most complete form in *Principia Mathematica*.[1] It may, however, be of some use to discuss certain of the distinctions which he made in the earlier work, and thereby show the difficulties in carrying out an analysis of classes. The difficulties in the earlier work were a sufficient reason for Russell to abandon, if at all possible, the assumption

[1] For Russell's theory of classes in *Principles of Mathematics*, see references in text. His later theory is in *Principia Mathematica*, I, 71 ff., I, *20. He gives a good, brief account in *Introduction to Mathematical Philosophy*, Chapter XVII. Discussions of his later theory may also be found in many modern books on mathematical logic.

In *The Philosophy of Bertrand Russell*, a clear presentation of Russell's definition is given by Ernest Nagel, pp. 323 ff., and a technical criticism of that definition is given by K. Gödel, pp. 133 ff.

Russell's claim that his theory 'eliminates' entities is criticized by W. V. Quine, *Mathematical Logic*, New York, 1940, pp. 119 ff. He cites further of his own works on the same point, p. 121n.

that classes were entities. The more technical details concerning classes cannot unfortunately be more than indicated here; they are significant, as Russell says, because they illustrate the importance of modern logic in investigating philosophical subjects. The more precise conditions that a theory of classes must meet are clearly shown only by means of symbolic logic and investigations into the foundations of mathematics. At the time Russell wrote problems had been raised by infinite classes, contradictions in the existence of certain classes, the null class, and the difference between a single-membered class and its only member. Russell found the work of logicians such as Peano and Frege of the most value to him in investigating classes, and classical logicians who had not known of such logical and mathematical considerations of little value.

In the *Principles of Mathematics*, Russell found it necessary to assume the existence of two entities 'class', the 'class as many', and the 'class as one'. Which one of these was a constituent of a given proposition depended upon the kind of statement being made about the class in question. Both entities were required because a class can be considered not only as all of the members of that class, but also as a unit, as a whole, a common property exemplified by its members. We cannot, moreover, dispense with the two entities 'class' and simply identify the class either with its members or with its determining characteristic. When the class is identified with its members, it is determined by an enumeration of those members, and is nothing other than those members. When it is identified with its determining characteristic, it is nothing but this characteristic, or concept.

Such an identification, Russell maintains, is not adequate. Identifying the class with its members, an extreme 'extensional' view, is not sufficient, since a class could be definitely determined only when all its members were enumerated, a procedure generally impractical, if not impossible, as, for example, with the class 'all mice' (66).[1] Identifying the class with its deter-

[1] Throughout this Part, references in the text by page numbers only are to the *Principles of Mathematics*.

mining concept is also not sufficient (the extreme 'intensional' view), for the same group of objects may have several different common properties.

> In order to perceive that *man* and *featherless biped* are not identical, it is quite unnecessary to take a hen and deprive the poor bird of its feathers. Or, to take a less complex instance, it is plain that *even prime* is not identical with *integer next after* 1. Thus when we identify the class with the class-concept, we must admit that two classes may be equal without being identical. Nevertheless, it is plain that when two class-concepts are equal, some identity is involved, for we say that they have the *same* terms. Thus there is some object which is positively identical when two class-concepts are equal; and this object, it would seem, is more properly called the class (68).

Any view of classes must in some way comprehend both the intensional and extensional approaches, since

> It is essential that the classes with which we are concerned should be composed of terms, and should not be predicates or concepts, for a class must be definite when its terms are given, but in general there will be many predicates which attach to the given terms and to no others (66).

Russell decides that if he must favour either view, he would incline toward the extensional view, because although it is true that classes can be determined by concepts, and some can only be determined in that way, yet we determine that two class-concepts are equivalent by finding that they determine the same object, or objects.

Although we cannot identify a class either with its members or its defining property and obtain an adequate analysis of 'class', it might be thought that *one* entity 'class' would be sufficient. But this, Russell maintains, is also not an adequate view, since one entity to comprehend both the reference to the class as composed of members, and as determined by a single property, would have to be an entity which is both one and many.

> If there is such an object as a class, it must be in some sense *one* object. Yet it is only of classes that *many* can be predicated. Hence,

if we admit classes as objects, we must suppose that the same object can be both one and many, which seems impossible.[1]

The employment of two entities, the class as many and the class as one, avoids this difficulty, but their use in turn brought further difficulties which were finally resolved only by the logical construction of classes. The class as many may be considered to be the numerical conjunction of the members of the class. Russell characterizes this conjunction:

> In a class as many, the component terms, though they have some kind of unity, have less than is required for a whole. They have, in fact, just so much unity as is required to make them many, and not enough to prevent them from remaining many. . . . When a class is regarded as defined by the enumeration of its terms, it is more naturally called a *collection*. I shall for the moment adopt this name, as it will not prejudge the question whether the objects denoted by it are truly classes or not. By a collection I mean what is conveyed by '*A* and *B*' or '*A* and *B* and *C*', or any other enumeration of definite terms. The collection is defined by the actual mention of the terms, and the terms are connected by *and*. It would seem that *and* represents a fundamental way of combining terms, and that just this way of combination is essential if anything is to result of which a number other than 1 can be asserted (69).

Russell's conception of the class as many is that it consists of an enumeration of terms; thus we enumerate *A*, and enumerate *B*, and so on, where *and* refers to the successive acts of enumeration.[2] But as Russell has pointed out, the reference to a class by intension, by means of the property common to all the members of the class, is also necessary; in the class as many there is no mention of the relation between the members and their common property by which we can also define that class.

Besides the class considered as many, then, we require also the class considered as one. We often speak of *a* class, as a class as an entity, as identical or different from some other class, as being a member of another class, in these cases it seems that

[1] *Principia Mathematica*, I, 72n.

[2] Compare this with Johnson's 'enumerative and', W. E. Johnson, *Logic*, I, 28, 122.

the class occurs as a term, as a single entity that can be made the subject of propositions about the class. An example from common speech may show this: (76) 'Classes of rational animals' considers the human race as one term, rather than as the many men referred to by 'all men'. Since 'all men' refer to men as many, the 'human race' if it denoted men as many would be identical with 'all men', but as it is not identical, we can infer a difference in the objects denoted. In the case of the 'human race', the object denoted is the class as one. Russell believed at this time that a symbol representing a class in symbolic logic must stand for the class as one, and that the epsilon relation of class membership held between a member of a class and the corresponding class as one. The class as one is a new single term, distinct from each of its members, and from all of them collectively; 'such a whole is completely specified when all its simple constituents are specified; its parts have no direct connection *inter se*, but only the indirect connection involved in being parts of one and the same whole' (140).

The two crucial instances of classes to which Russell applied his theory were the null class and the single-membered class. The null class, the class with no members, is important for logical and mathematical purposes, and besides, is met with in common speech when one refers either intentionally or through ignorance to a class with no members. Thus one can speak of 'ghosts' or of 'those elephants native to America', although the classes referred to will be null classes. We cannot, however, extend Russell's notion of the class as many and the class as one to cover the null class, and some other explanation must be found. Russell believes that in the case of the null class, although the class as many and the class as one do not exist, all formal purposes can be served by using the 'class of all null class-concepts' (75). This is a class in Russell's sense, whose members are class-concepts, and of whose existence there is no doubt since the properties or characteristics determining a null class are as much properties or characteristics as those determining any other class.

The single-membered class is the class with only one member, and occurs in such examples as 'living ex-Presidents of the United States' and 'the novelist in a class by himself'. In the *Principles of Mathematics* Russell at first considered this class to be identical with its only member. Considering the class as many, this conclusion seems reasonable, since that class is a numerical conjunction of terms; when the class has only one member that member will be the class as many. For the class as one, the situation is not so clear, but Russell considers it 'self-evident' (77) that a whole composed of one term only is that one term. A class with several terms when considered as one is a term distinct from its parts, but completely specified by them; when there is only one term the whole will be that one term. I shall discuss shortly, however, that other considerations forced Russell to modify his conception of this class even in the *Principles of Mathematics*.

Not only must the theory of classes I have been presenting account for the null class and the single-membered class, but it must also be so formulated as to avoid introducing contradictions. These contradictions, some of which Russell himself discovered in the course of developing his logic, led him first in the *Principles of Mathematics* to modify the theory I have so far discussed, and finally, were an important reason for its abandonment altogether. They are the contradictions which led also to the development of the 'theory of types'. One important contradiction, especially for the theory of classes, is that which occurs if we assume that a class can be significantly asserted to be a member of itself. Unless this assumption can be excluded, the theory of classes will be found to contain the contradiction, and thus fundamentally weakened.

To avoid this contradiction, Russell developed in the *Principles of Mathematics* a preliminary formulation of the theory of types (Appendix B), and adopted various modifications of his original theory. As a first step Russell denied the existence of the class as one in those cases where its existence would lead to a contradiction (104). In those cases where the existence of the

class as one is denied, we cannot use 'all' to refer to the whole composed of the terms of the class, but must use 'all' in the distributive sense, referring to the class as many. Originally, the class as one had been introduced because of the apparent need to treat a class as one term; but the contradictions showed that in spite of the usefulness of the class as one, in some cases it could not exist. It was necessary then to find a way in which the class as many could be adequate to serve all the purposes for which the class as one had been considered essential, and Russell believed that he had found such a way (132).

The notion of the class as many is also found to need revision Originally Russell maintained that the class as many was merely a numerical conjunction of terms, yet the theory of types was found to require that classes be of a 'higher type' than their members. If the class as many is a numerical conjunction of terms, how can it be of a higher type than its members? (131). It must, apparently, be a separate entity, but the original conception of the class as many had been made to avoid this separation. The theory of types, besides intuitive arguments, also leads Russell to abandon his identification of the single-membered class with its single member. The class must be of a higher type than its member, and consequently is not identical with it (131, 513).

The final version of the theory of classes in the *Principles of Mathematics*, arrived at after Russell had considered Frege's views (518), is a considerable modification of his first view. The class as many, which is defined by a propositional function, is always present, while the class as one may or may not be present, and is apparently of secondary importance. For the single-membered class and the null class, Russell modifies Frege's notion of the set of values for which a propositional function becomes a true proposition (511), so that those classes are identified with the set of values of their defining functions. The set of values is an object, but an object of a different type from the terms of the class. The theory of classes presented in the *Principles of Mathematics* was not entirely clear to begin with,

and Russell's modifications of it to meet specific difficulties hardly makes it more lucid. But we can find in his last analysis of the null class and the single-membered class an indication of the direction his final analysis of classes takes, and I should like now to consider that later theory.

In the theory of classes as it is presented in *Principia Mathematica*, Russell has made the use of propositional functions of central importance. The effectiveness of propositional functions for this purpose can be seen if it is recalled that in speaking of classes we are concerned with a group of objects, alike in the possession of a common property. A propositional function can be assumed to represent the common property, and the values which satisfy that function will be the objects possessing that property. The class, however, cannot be identified with the propositional function itself, for the same reason that the class could not be identified with the common property or concept determining the class, nor can it be identified with the values satisfying the function, since then the class would be identical with its members. Further, if we assume that classes are entities defined by propositional functions in some way, but not identical with them or their extension, then we are faced with the same problems as were encountered in Russell's earlier theory; for we would have to determine whether the entity were 'one' or 'many', and whether or not it could avoid the contradictions which faced the earlier theory.

Instead of using propositional functions in any of these ways, we may by their means treat 'class' as an incomplete symbol, defining it only in use. 'Class' then can be analysed by means of logical constructions. Russell finds that such definitions serve all the purposes for which 'class' is required, and avoid the necessity of assuming that 'class' denotes an entity. Instead of defining, for example, 'class', Russell defines 'x is a member of class a'. The statement equivalent to 'x is a member of class a' will correspond to a proposition which contains no entity 'class', but only entities already accepted such as properties and their instances. In this example, 'x is a member of the

class a' is equivalent to 'x is a value that satisfies f', where f is the propositional function that is usually said to determine the class a. The fact that the statements are equivalent can be more readily understood if it is remembered that a value satisfying a propositional function has the property that the function expresses, and hence would commonly be said to be a member of the class determined by that property or function. The reader should not be misled by the fact that symbols for classes have been used, apparently denoting entities 'classes'. It is customary to introduce such symbols, so that instead of using the cumbersome symbolic expression for 'values satisfying the propositional function f' a more convenient expression can be used. The fact that these symbols are read 'class' should no more be assumed to imply that Russell maintains that class symbols denote entities, than that the employment of symbols for numbers and objects described implies that they denote particular kinds of entities. Logically, according to Russell, such simple symbols could be dispensed with.

The preceding example is not sufficient, of course, to show that a definition in use of 'class' is adequate. I shall discuss only a few statements concerning classes, and refer the reader to Russell's writings for a thorough discussion of his definition of 'class.' The notion of 'identity' between classes is a problem for the theory of classes, since it is desirable, as Russell says, to consider two classes identical if they have the same members, but at the same time distinguish the classes if they are determined by different properties. If a class is considered to be one entity, it would be difficult to account for 'identity' in this sense between classes. Using propositional functions, we find that the identity between classes can be expressed as the formal equivalence of two propositional functions, avoiding the use of 'class' at all. The example which Russell uses of identical classes is that the 'class of men' is identical with the 'class of featherless bipeds'. In this example, the functions 'man' and 'featherless biped' are formally equivalent since every value that satisfies one satisfies the other. The use of 'identity' between

classes is adequately covered because both functions will be satisfied by the same values, but the two functions, although equivalent, will not be identical. Similarly, if two propositional functions are not formally equivalent, they will be said to determine different classes.

The null class and the single-membered class will be treated no differently from other classes. Those propositional functions that are satisfied by no values will be said to determine the null class; and those propositional functions satisfied by only one value will be said to determine a single-membered class. The discussion of Russell's later theory of classes so far given will hold for these classes, because they are distinguished from classes with more members only in that the function determining the null class is satisfied by no value, and that determining the single-membered class is satisfied by only one value. There is no problem of identifying the single-membered class with its only member, since that class as an entity does not exist. The property determining that class is a propositional function, the member of the class is the sole value satisfying that function. The contradictions which were met in the earlier theory of class are not found here, since the theory of types places restrictions upon what can significantly be a value of a propositional function, thus preventing the formation of 'classes which are members of themselves'.

This theory of the nature of 'class' not only shows that it is possible to dispense with the assumption of an entity 'class', but also provides a more adequate analysis of the notion of class than did Russell's earlier theory. His first theory suffered from the employment of two vaguely defined entities, and could not provide a uniform explanation for all classes, requiring that exceptions be made to meet special problems. Thus when Russell discovered the contradiction of classes which are members of themselves, he at first attempted to avoid this contradiction by dispensing with the class as one where it led to difficulties. The later theory can be applied to all classes, and besides, adequately cover the usual meaning of class. The

definition in use of 'class' is an additional example of the success of the employment of logical constructions. By means of logical constructions, Russell has found it unnecessary to assume the existence of entities corresponding to numbers, objects described, and now classes. We have not, of course, eliminated the existence of universals, or properties, but only that of classes; but this is a considerable advance over assuming that not only properties, but two entities, the class as many and the class as one, exist.

CHAPTER III

THE PROBLEM OF THE EXTERNAL WORLD

THE problems I considered in the preceding Chapter were specialized logical or mathematical problems for whose solution Russell developed a technique which he was to find of great usefulness in attacking more extensive problems. In this Chapter, I shall discuss several general philosophical problems with which Russell is greatly concerned in his later writings. Critically examining common-sense beliefs will reveal philosophical problems, Russell believes, and lead us to determine the extent to which such beliefs are sound, and what elements are really involved in them. The primary problem which he first found in this way is the common-sense belief in permanent material objects, or 'matter', and it is the inquiry into the possible justification of this belief that occupies an important place in his philosophical interests. In *The Problems of Philosophy* (1912) we find a forceful and over-simplified expression of this approach to philosophy, and the problem of material objects which he determines through that approach. He maintains it is necessary to investigate ordinary beliefs to find the extent to which they are sound, which of them, if any, are 'certain', which are only instinctive beliefs, or worse, prejudices. Russell compares himself to Descartes in wishing to find which of our knowledge is 'certain', which 'doubtful'. In addition, it is also valuable to examine the soundness of scientific knowledge, and the basis upon which that knowledge rests. In his later writings, especially *Our Knowledge of the External World* (1914), *The Analysis of Matter* (1927), *Philosophy* (1927), *An Inquiry into Meaning and Truth* (1940), and *Human Knowledge* (1948), his first rather crude

expression of the problem becomes more refined and transformed into one version of the problem which concerns so many philosophers today, the foundation and basis of empirical knowledge, and the relation of perception to that knowledge.

In Part I, after first briefly presenting Russell's views in *The Problems of Philosophy*, I shall summarize the criticisms of common sense and scientific knowledge and the problems to which that criticism gives rise as developed principally in *Our Knowledge of the External World*, several other papers written about the time of *Our Knowledge of the External World*,[1] *The Analysis of Matter*, *An Inquiry into Meaning and Truth*, and *Human Knowledge*. Russell also formulated a philosophy of logic and an analysis of propositions which he called the philosophy of 'logical-atomism' that led him by a somewhat different route to the same problems. I shall present these views in Part II. It is a logical doctrine which can be found in writings around 1918, and for a period thereafter, most fully expressed in the lectures, 'Philosophy of Logical Atomism' (1918–19). Finally, in Part III, I shall discuss the need for an 'interpretation' of science if we are to examine the question of the validity of scientific knowledge. Russell's views on this point are found chiefly in *The Analysis of Matter* and *Human Knowledge*.

PART I

HARD vs. SOFT DATA

It is significant that the first sentence of *The Problems of Philosophy* is 'Is there any knowledge in the world which is so certain that no reasonable man could doubt it?' (9). It is in this spirit that Russell approaches philosophy in that book, trying to find the certain elements in our common-sense knowledge,

[1] These papers include *Scientific Method in Philosophy*, Oxford, 1914; 'The Ultimate Constituents of Matter', *Monist*, XXV (1915), 399–417; 'The Relation of Sense-Data to Physics', *Scientia*, No. 4 (1914). These were later reprinted in *Mysticism and Logic*, New York, 1918. Hereafter I shall merely refer to *Mysticism and Logic* without distinguishing between the three papers.

and the extent to which it is reasonable to continue to believe the remaining elements. The common-sense belief that Russell is most interested in is that in the existence of permanent material objects, or matter. Russell employs familiar epistemological arguments in an attempt to show that we do not directly perceive such objects, but must obtain our knowledge of them in some way from our own sensations, or sense-data. The existence of sense-data cannot be doubted, but that of material objects can.

> Before we embark upon doubtful matters, let us try to find some more or less fixed point from which to start. Although we are doubting the physical existence of the table, we are not doubting the existence of the sense-data which made us think there was a table; we are not doubting that, while we look, a certain colour and shape appear to us, and while we press, a certain sensation of hardness is experienced by us. . . . In fact, whatever else may be doubtful, some at least of our immediate experiences seem absolutely certain.[1]

Starting from this 'solid basis' do we have 'any reason for regarding [our sense-data] as signs of the existence of something else, which we can call the physical object'?[2]

Some of our 'immediate experiences' Russell has called 'absolutely certain'. However, the sense-data themselves are not certain in the sense that propositions may be certain, or absolutely true, but rather just *are*. 'A particular patch of colour which I see, for example, simply exists: it is not the sort of thing that is true or false' (178). Judgments about these sense-data, however, may be 'self-evident' in the more usual sense of the word. These 'self-evident truths' are of two kinds: 'First, there is the kind which simply asserts the *existence* of the sense-datum, without in any way analysing it. . . . The other kind arises when the object of sense is complex, and we subject it to some degree of analysis. If, for instance, we see a *round* patch of red, we may judge "that patch of red is round" '

[1] *The Problems of Philosophy*, London and New York, 1912, p. 27.

[2] *Ibid.*, p. 30. For the next few pages, references in the text will be to *The Problems of Philosophy*.

(178–9). Russell distinguishes between degrees of self-evidence, the highest degree of self-evidence gives an 'absolute guarantee of truth'. Truths about sense-data possess this highest degree because we can have 'acquaintance' with the fact which corresponds to the particular truth, namely the particular sense-datum concerned (212). Starting from our own sensations, either sense-data or propositions about them, the problem is to determine the grounds for the belief in the existence of permanent material objects.

The solution of this problem which Russell maintained in *The Problems of Philosophy* is a form of the causal theory of perception. Material objects are the causes of our sense-data, are the reason why there is agreement among observers as to what they see, are the reason for whatever stability and orderliness exists among our sensations. In fact, it is the ease with which the belief in material objects explains this orderliness and stability that is one of the chief reasons he finds in favour of the causal theory. But Russell is looking for more certain grounds than usefulness and plausibility. However, the best Russell can do toward establishing this belief on other grounds is to find 'this belief ready in ourselves as soon as we begin to reflect: it is what may be called an *instinctive* belief' (37). It does not seem to be a self-evident belief like one about sense-data, but we are justified in accepting this belief, Russell feels, since it is a belief we hold strongly, and 'there can never be any reason for rejecting one instinctive belief except that it clashes with others; thus, if they are found to harmonize, the whole system becomes worthy of acceptance' (39).

Although we are justified in believing that material objects exist, we can infer little as to their nature. We can assume that if there is a difference between sense-data, there must be some corresponding difference between material objects. If two sense-data are related to each other as right to left, then we can suppose that the corresponding objects are related in a similar relation. But the intrinsic nature of these objects seems to be something we cannot know, at least by the senses. A red

sense-datum has the quality red to our senses, but it is unlikely that the object itself is red in the same sense, since physics tells us that the red sense-datum is the result of a complicated process involving among other things light and a sense-organ (54, 59). The discussion on these points is brief in *The Problems of Philosophy*, hardly more extensive than my presentation of them. It would be unfair to criticize this book too harshly, since it was not intended to be a detailed work. Russell wrote most of his later works about points only briefly treated in *The Problems of Philosophy*, in many cases modifying the views which he expressed there. In the remainder of this Part I shall discuss his more comprehensive treatment, in works following *The Problems of Philosophy*, of the distinction between basic and inferred knowledge, and his belief in the desirability of grounding our knowledge on basic knowledge. As in *The Problems of Philosophy*, it is largely the problem of the existence of material objects to which these distinctions are applied. His solution to the problem I will leave to the following Chapter.

In *Our Knowledge of the External World*, the common-sense knowledge that forms the material for philosophic inquiry Russell divides into several kinds, knowledge that we have obtained from our own experience of objects and events—such as towns, people, and their actions—knowledge that we have by testimony of objects and events that have not been experienced by us, and finally scientific knowledge which systematizes our more particular knowledge by means of inclusive generalizations. This basic material for examination forms the 'data' for philosophic problems, as the 'data' for a science, like psychology, would consist in human actions and behaviour.[1] It should be noted that Russell uses 'data' in different senses in his writings. One use is that which I have just mentioned, the material from which any inquiry starts. Epistemological

[1] *Our Knowledge of the External World*, first edition, London and Chicago, 1914, p. 65. Unless otherwise noted all references will be to the first edition of this work.

data are further distinguished into 'hard' and 'soft' data, and 'data' then comes to mean simply 'hard data'. Russell makes this distinction explicit in the *Monist* for 1914, where he says of data in the second sense: '. . . what we have hitherto called data will be more fitly called premises.'[1] In *Human Knowledge* he says, 'There is thus a distinction between beliefs that arise spontaneously and beliefs for which no further reason can be given. It is the latter class of beliefs that are of most importance for theory of knowledge, since they are the indispensable minimum of premises for our knowledge of matters of fact. Such beliefs I shall call "data" '.[2] Normally when Russell speaks simply of 'data' he means data in this latter sense. I shall follow this same practice in my discussion.

Our original data is not all of the same degree of certainty; it can be classified in respect to its relative certainty or uncertainty, and the degrees of certainty of the different facts we know are themselves part of the data of common knowledge. Not only do we begin with various facts of perception, or of history, or of science, but also with the knowledge, though not necessarily very precise, of the degree of confidence we can have of the truth of that knowledge. That data which is certain Russell calls 'hard data', that which after examination is found to be not certain, is 'soft data'.[3]

> I give the name 'data' or rather 'hard data' to all that survives the most severe critical scrutiny of which I am capable, excluding what, *after the scrutiny*, is only arrived at by argument and inference.[4]

The part of our knowledge which can be called 'hard data' is further characterized as that data which is certain, self-evident, where 'self-evident' means believed on its own account,[5] or

[1] 'Definitions and Methodological Principles in Theory of Knowledge', *Monist*, XXIV (1914), 586.

[2] *Human Knowledge*, New York, 1948, p. 166.

[3] *Our Knowledge of the External World*, p. 70.

[4] 'Professor Dewey's "Essays in Experimental Logic",' *Journal of Philosophy*, XVI (1919), 21.

[5] *Our Knowledge of the External World*, p. 68.

'known otherwise than by inference'.[1] Knowledge which is not found to be certain is 'soft data', even though at first sight it may seem as certain as hard data, upon examination such knowledge is found to be really 'inferred' or 'derivative' knowledge. Soft data are not certain, Russell believes, because they are based upon some other knowledge, and the inference from the prior knowledge to them might be erroneous.

The distinction between hard data and soft data corresponds in *Our Knowledge of the External World* to that between 'primitive' and 'derivative' knowledge.[2] Hard data, or primitive beliefs, are believed on their own account, while derivative, or 'inferred', beliefs are those 'we only believe because of something else from which [they have] been inferred in some sense, though not necessarily in a strict logical sense'.[3] Most common-sense knowledge, it turns out, is really derivative knowledge, thus the real shape of a perceived object is inferred from our particular perceptions; our knowledge of a person's feelings is an inference from his observed actions and our experience of him and human nature; and, of course, we have knowledge which we have obtained by conscious inference from previously substantiated beliefs. For the present, we can interpret 'inference', as Russell is using it in discussing derivative beliefs, in a sense broad enough to cover all cases of obtaining knowledge from previously established knowledge.

Which of our knowledge, now, is 'hard data'? How much of our knowledge can resist the severest 'critical scrutiny'? Russell's

[1] *Journal of Philosophy*, 1919, 22. Hard data is also later defined as 'those matters of fact of which, independently of inference, we have a right to feel most nearly certain' (*Human Knowledge*, p. 171), and a 'datum' is a proposition 'which has some degree of rational credibility on its own account, independently of its relations to other propositions' (*Ibid.*, p. 384). And: 'The essential characteristic of a datum is that it is not inferred' (*An Inquiry into Meaning and Truth*, New York and London, 1940, p. 155. See also *ibid.*, p. 17).

[2] *Our Knowledge of the External World*, p. 68.

[3] *Ibid.*, p. 68. The correspondence between hard data and primitive beliefs, and soft data and derivative beliefs, is subject to the qualification mentioned below, p. 96.

emphasis on critical scrutiny indicates that we determine what is certain knowledge, or hard data, by the indirect process of finding what is not soft data. If we examine any of our beliefs, and can detect no process of inference, then that belief is classed as certain. There are several large classes of beliefs which at first sight appear to be certain but upon examination are found to be really soft data. Our knowledge of material objects, or of the existence of other persons' minds is not at all primitive or certain knowledge. Our belief in material objects is based on generally constant correlations between perceptions which we receive from one sense (e.g. sight) and those we receive by other senses (e.g. touch), or between our present perceptions and those obtained as a result of some kind of action. Thus if we are seated around what is called a table we expect certain tactile sensations from reaching out and touching it, and expect a certain series of visual impressions if we get up and walk around it. Such groupings of perceptions is the basis of our belief in permanent material objects; but such beliefs are not certain since the correlations may not invariably hold. The familiar arguments of the illusions of the senses are examples of perceptions with which we expect other perceptions to be correlated, but where in fact the correlation breaks down. The belief in material objects Russell calls an inference, since it is a belief not founded merely on the isolated immediate perceptions themselves, but is a result of correlations between perceptions. The existence of other persons' minds is also an inference; for we infer that they have thoughts and feelings analogous to our own on the basis of their actions or words or expressions. Our immediate perceptions, Russell believes, are not erroneous, in fact in *Our Knowledge of the External World* (71) they are 'luminously certain'; error occurs in beliefs that are inferences from these perceptions, when a perception whose occurence is expected because of its usual correlation with a present perception fails to appear. Even if the error involved in expecting such an occurrence is infrequent, the belief cannot be classed as certain, or among our primitive data.

The upshot is that what seems like perception of an object is really perception of certain sensible qualities together with expectations of other sensible qualities—the commonest case being something visual which arouses tactual expectations. . . . Hence we conclude that we have to do with a correlation which is usual but not invariable, and that, if we wish to construct an exact science, we must be sceptical of the associations which experience has led us to form, connecting sensible qualities with others with which they are often but not always combined.[1]

Hard, or primitive, data include then little more than

. . . coloured shapes which move, . . . noises, smells, bodily sensations, the experiences which we describe as those of touch, and so on. There are relations among these items: time-relations (earlier and later) among all of them, and space-relations (up-and-down, right-and-left, and the relations by which localization in the body is effected) among many of them. There are recollections of some of these things. . . . There are also expectations; by this I mean something just as immediate as memory.[2]

In addition, universals and general propositions of logic are often included by Russell among the kinds of hard data.[3] The most important large class of hard data for the purposes of establishing empirical knowledge is that of our own perceptions.[4] It is, however, not clear just what constitutes a particular percept, what precisely is this certain base for our knowledge. In the above quotation Russell gives merely a general statement referring to 'sensations'; in *Our Knowledge of the External World* again reference is made simply to the 'immediate objects of sense' and 'facts of sense' (70). In the *Journal of Philosophy* for 1919 (p. 24) basic data includes particulars and facts such as 'what is seen and heard . . .'. Later, however, in *The Analysis of Matter* there is a more extensive discussion on what constitutes

[1] *The Analysis of Matter*, London and New York, 1927, p. 182.

[2] *Ibid.*, pp. 180–1.

[3] *The Problems of Philosophy*, Chapter X; *Our Knowledge of the External World*, pp. 70–1; *Monist*, 1914, 585; *Journal of Philosophy*, 1919, 24.

[4] Data from memory are also important as a basis for inferred beliefs. The consideration of such data would, I believe, not appreciably affect my discussion, and consequently I am not considering them in this work. The reader may find extensive discussions of memory in many of Russell's works.

a perception. Russell finds that sensations are not clearly differentiated, that perceptions are accompanied by 'interpretations' of varying degrees, where an interpretation may include the assumed relation of the perception to some 'object', or the correlation of sensations of various kinds with the original sensation. How much of the interpretation, if any, should be included in the original hard datum, how much among inference from that datum?

> Perception must include those elements which are irreducibly physiological, but it need not on that account include those elements which come, or can be made to come, within the sphere of conscious inference. When we hear (say) a donkey braying, we are quite conscious of inference from the noise of the donkey, or at any rate we can easily become conscious of it. I should not, therefore, in this case, include anything else of the donkey with the perception, but only the noise . . . I should therefore say that a great deal of the interpretation that usually accompanies a perception can be made conscious by mere attention, and that this part ought not to be included in the perception. But the part which can only be discovered by careful theory, and can never be made introspectively obvious, ought to be included in the perception. Perhaps the line between the two is not so sharp as could be wished. . . .[1]

Apparently, then, there is no sharp distinction between hard data and derivative knowledge; what is a datum must be ascertained by reflection, by eliminating all derived knowledge and up to the point suggested by the preceding quotation all inferences. It is what remains after such inferences have been eliminated:

> It is also clear that what we *think* we observe is usually much more than what, after closer attention and more analysis, we find we really did observe—because habitual inferences become unintentionally mixed up with what was actually observed. Thus the conception of a 'datum' beomes, as it were, a limiting conception of what we may call scientific common-sense.[2]

[1] *The Analysis of Matter*, p. 189. See also *Philosophy*, New York, 1927, p. 204.

[2] *Journal of Philosophy*, 1919, 21. See also *The Analysis of Mind*, London and New York, 1921, p. 298.

Although in *The Analysis of Matter* Russell does not maintain that *no* elements of interpretation, or inference, should be included among data, in *An Inquiry into Meaning and Truth* and *Human Knowledge* he apparently wishes to avoid including any such elements. Thus he distinguishes between 'sensation' and 'perception', and maintains that only the former can constitute 'data'. Briefly, sensation includes no interpretation, no inference, while perception is sensation expanded by interpretation.[1] The sensational 'core', Russell maintains, cannot be illusory.

> Every sensation which is of a familiar kind brings with it various associated beliefs and expectations. When, say, we see and hear an airplane, we do not merely have the visual sensation and the auditory sensation of a whirring noise; spontaneously and without conscious thought we interpret what we see and hear and fill it out with customary adjuncts. To what an extent we do this becomes obvious when we make a mistake—for example, when what we thought was an airplane turns out to be a bird.[2]

What is illusory, as far as illusions of the senses are concerned, is our interpretation, which, unlike the sensation, can be mistaken.

> . . . we cannot admit as data all that an uncritical acceptance of common sense would take as given in perception. Only sensations and memories are truly data for our knowledge of the external world. We must exclude from our list of data not only the things that we consciously infer, but all that is obtained by animal inference, such as the imagined hardness of an object seen but not touched.[3]

Elsewhere in *Human Knowledge*, however, Russell speaks of 'perception' and 'percepts' as data.[4] This is not necessarily inconsistent with the view in the preceding paragraph, since in discussing 'percepts' Russell is not giving an extensive dis-

[1] *Human Knowledge*, p. 169.
[2] *Ibid.*, p. 167.
[3] *Ibid.*, p. 170. There is a similar view in *An Inquiry into Meaning and Truth*, p. 154.
[4] *Human Knowledge*, pp. 8, 203 ff.

cussion of data and does not need to employ 'percept' in the strictest sense of the word. I shall assume that the preceding paragraph represents Russell's real views in *Human Knowledge*, and that he uses 'perception' as referring to that which, in a loose sense, comes through the senses and is the basis for our empirical knowledge.

The view of 'data' in *An Inquiry into Meaning and Truth* and *Human Knowledge* might seem to suggest that view of data as 'certain' which I have quoted from some of Russell's earlier works. This is, however, not the case, for he specifically mentions that data may be uncertain.[1] His examples are cases of faint perception, such as the sound of a distant airplane, which we are sure we hear at one point, but which probably passes out of the range of our hearing without our being able to determine precisely at what point we have ceased to hear it.[2] One might question that if it is only sensation (and not perception) which is a datum, then whether or not whatever faint sensation we have can be determined to be caused by an airplane or our imagination would be irrelevant, since this latter determination is one concerned with the correctness of our perception, that is, with the inferences from our sensations. However, I shall discuss these points at greater length later in this section.

I have listed as included among 'data' simply 'perceptions' (or 'sensations') and have not distinguished them from 'facts of sense' or propositions concerning sensations. Russell's statements are not clear on this point. Thus the quotations given mention perceptions and sensations, but in the *Monist* for 1914 (p. 585) he says, '. . . a given person's data are those *particulars* with which he is acquainted. . . . Again, *perception of facts* . . . must be included among data . . .'. A similar view can also be found in *Mysticism and Logic* (147) and *Our Knowledge of the External World* (53). In *An Inquiry into Meaning and Truth*

[1] *Human Knowledge*, pp. 173, 393 ff.; *An Inquiry into Meaning and Truth*, p. 155.

[2] *Human Knowledge*, p. 393; *An Inquiry into Meaning and Truth*, p. 155.

and *Human Knowledge*, although Russell speaks of 'sensations' as data, we find that he sometimes speaks of a *proposition* as being a datum.[1] We can reconcile including 'facts' among data by accepting the point of view indicated in *Mysticism and Logic* (147). In that passage, Russell maintains that we are acquainted with perceptions, which are data, and also with facts of perception, which are facts with particular perceptions as terms. Even in the case of facts of perception, it is perceptions upon which they ultimately rest, and we can disregard such facts in discussing data. The relation between sensations and propositions about them is not so simple to reconcile. I shall return to this point later.

The fact that we can never determine precisely what is a datum and what is really inference does not invalidate the procedure of trying to find the most certain parts of our knowledge, and then using those certain elements to justify the remainder of our knowledge. The more precisely we establish what is data and what inference, the more definitely will we know what elements are certain in our knowledge; the remaining area of uncertainty will be the limiting point beyond which our justification of knowledge cannot extend. But the fact that we reach a limit beyond which we cannot—at present—penetrate does not make the work to that point useless, the greater clarity that we have about the basis of knowledge than we do on the common-sense level is a definite accomplishment and all to the good.

The difficulty of distinguishing between data and inferences might lead to the view held, Russell maintains, by such philosophers as Hegel and Dewey that there is *no* distinction between the two.[2] One reason which might be given for denying the distinction is that the understanding and interpreting of data appears to be carried out against a background of previous knowledge and experience. Even so simple a statement

[1] *Human Knowledge*, p. 392; *An Inquiry into Meaning and Truth*, pp. 17, 156, 171 ff.

[2] *Human Knowledge*, pp. 391 ff; *An Inquiry into Meaning and Truth*, p. 154.

as 'that is a stick' assumes a framework within which a piece of wood is recognized as a stick, rather than as a club, or an old chair leg, or a discarded tool handle. This is even more apparent in complex cases such as 'that is an eclipse of the moon' where the meaning of 'eclipse' depends on elaborate scientific knowledge. Further, it may be difficult to find examples of data which are accepted only as data, without additional grounds for them being given by other beliefs. It may possibly be the case that no proposition can be found certain or uncertain in isolation from other propositions, but that our knowledge forms an interconnected whole within which the certainty of any proposition can be determined only by its relation to other propositions. However, Russell maintains, granting that these points are to some extent valid, it is still necessary to have some propositions, which, whether or not they can be given greater certainty by other propositions, are themselves data in that they have 'some degree of independent credibility'. Without 'data' we could have no knowledge that could not be logically derived from our previous knowledge, as we surely seem to have.

> . . . it may be possible in the end to arrive at a body of interconnected propositions having, as a whole, a very high degree of credibility. Within this body, some are only inferred, but none are only premises, for those which are premises are also conclusions. The edifice of knowledge may be compared to a bridge resting on many piers, each of which not only supports the roadway but helps the other piers to stand firm owing to interconnecting girders. The piers are the analogues of the propositions having some intrinsic credibility, while the upper portions of the bridge are the analogues of what is only inferred. But although each pier may be strengthened by the other piers, it is the solid ground that supports the whole, and in like manner it is intrinsic credibility that supports the whole edifice of knowledge.[1, 2]

[1] *Human Knowledge*, pp. 395–6. See Russell's 'Reply to Criticisms', in *The Philosophy of Bertrand Russell*, pp. 711 ff., for a similar view.

[2] Reichenbach in *The Philosophy of Bertrand Russell*, pp. 50 ff., also criticizes him from a similar point of view, but Russell presumably would answer his criticism as he answers that above.

Knowledge other than our primitive knowledge, or hard data, is inferred, or derived, from that primitive knowledge. So far I have not distinguished between different kinds of inference, but Russell finds various senses in which we can speak of knowledge being 'derived'. The clearest examples of derivative knowledge are those obtained by conscious inferences, the most difficult examples to distinguish are those that we can only become conscious of by close examination of our knowledge, such as the examples I mentioned earlier of the inference of the 'real' shape and size of a perceived object perceived from an unusual position, and the inference of a person's feelings from the expression of his face. Any bit of knowledge is derivative knowledge, or soft data, which is believed, as Russell says,[1] 'because' of something else from which it is inferred. 'Because' may mean either that it is believed because it has been inferred in a strictly logical sense, or that it is believed because of habit or psychological association. In *The Analysis of Matter*, Russell also distinguishes various kinds of inference.[2] The earliest stage of inference is 'physiological', the physiological response 'that governs all animal reactions, and a large part of man's reactions'.

> The next stage is where there is an actual passage from one belief to another, but the passage is a mere occurrence, not a transition motivated by an argument. In this case, the transition is usually caused by a physiological inference. Then there is inference based upon some belief; but even then the belief . . . may not logically warrant the inference. . . . Lastly there is valid inference by means of a true principle—but of this I cannot give an indubitable instance.[3]

The beliefs which Russell characterizes in *Our Knowledge of the External World* as those which 'we only believe because of something else from which it has been inferred in some sense, though not necessarily in a strict logical sense' (68), are later, in *The Analysis of Matter*, classed as beliefs that are the result

[1] *Our Knowledge of the External World*, p. 68.
[2] *The Analysis of Matter*, pp. 190 ff.
[3] *Ibid.*, p. 190.

of 'physiological inference'. The 'because' in the preceding quotation means that the beliefs have arisen through habit or psychological association, the example which Russell gives is that we physiologically 'infer' the real shape and size of a perceived object from the apparent size and shape. In *Philosophy* the notion of 'physiological inference' is discussed more fully, elaborated by a psychological theory which Russell maintains of habit and association. According to that theory we learn to perceive physical objects by experiencing correlations between the sensations given by different senses, in this manner determining certain habits. In the case of the real shape and size of a perceived object, we have experienced certain visual sensations of an object, and have further experienced that under certain conditions these will be followed by certain tactual sensations. As we move, our sensations will be followed by others in a familiar manner, and at some one position we will obtain the sensations which we have learned to call the ones from the 'real' object. Once we have established this correlation, the process from any one of the visual sensations of inferring the 'real' shape is almost instantaneous, and we are not conscious of the correlation which we have established by experience. But such beliefs, 'inferred' beliefs to Russell, are not certain since they are founded on this experienced correlation which may not hold—e.g. if the visual sensation of the object is only a dream. He defines this type of inference:

> Physiological inference, in its simplest form, means this: given a stimulus S, to which, by a reflex, we react by a bodily movement R, and a stimulus S' with a reaction R', if the two stimuli are frequently experienced together, S will in time produce R'.[1]

This view will also be found in *Human Knowledge*, where, however, Russell calls this type of inference 'animal inference'. The 'spontaneous' interpretation of sensations is one case of animal inference, in which expectations of qualities other than those directly perceived is associated with the received sensation. Thus we imagine that a table which we see will also

[1] *Philosophy*, p. 13; see also *ibid.*, pp. 80 ff.

feel hard if it is touched. 'Animal' inferences are the basis of many common-sense generalizations, and ultimately of many scientific ones.

> In animal inference, the percept *A* causes the idea of *B*, but there is no awareness of the connection; in scientific inference (whether valid or invalid) there is a belief involving both *A* and *B*, which I have expressed by '*A* is a sign of *B*'. It is the occurrence of a single belief expressing a connection of *A* and *B* that distinguishes what is commonly called inference from what I call animal inference. But it is important to notice that the belief expressing the connection is, in all the most elementary cases, preceded by the habit of animal inference.[1]

It will be found that though 'animal' or 'physiological' inference can be distinguished from conscious processes of inference, it is still necessary to determine the logical basis for such inferences.

Our knowledge, then, can be distinguished into 'data', that part of it which is certain, and 'derivative' or 'inferred' knowledge in any sense of the word, which includes the larger part of ordinary empirical knowledge. 'Inferred' knowledge appears valid to a large extent, and there is no question of the fact that everyone does consciously or unconsciously make such inferences; the problem with which Russell is concerned is to find the grounds and the extent to which such inferences are justified, and in particular, the inferences to permanent material objects and to the matter of physics. The reader may very well be puzzled by Russell's use of 'justify', and I shall try later to explain what I believe that he means; for the present, I shall merely follow his exposition. Starting from our certain knowledge of the world Russell wishes to see to what extent our derivative beliefs about material objects or matter can be justified so that, to counteract our normal unquestioning acceptance of those beliefs we will have valid reasons for the degree of certainty that these beliefs should possess. Thus as he says: 'Can the existence of anything other than our own hard

[1] *Human Knowledge*, p. 186; see also *ibid.*, pp. 167 ff., 182 ff.

data be inferred from the existence of those data?'[1] And the somewhat different problem: 'Can we know that other objects, inferable from objects of sense but not necessarily resembling them, exist either when we are perceiving the objects of sense or at any other time?'[2] In *The Analysis of Matter* Russell gives the following statement of his problem:

> The most important inference which science takes over from common sense is inference to unperceived entities. . . . This belief in the permanence of perceived objects has gone through all stages from physiological inference to advanced scientific or philosophical theory; the inquiry into its justification is the central problem in the analysis of matter, philosophically considered.[3]

And in a paper written in 1919 he says:

> The problem with which I am concerned is this: Enumerate particulars in the world and facts about the world as long as you can; reject what you feel to be doubtful; eliminate what you see to be inferred. There then remains a residuum, which we may call 'data'. . . . They are a certain collection of particulars and facts, and they are the total store from which, at the moment, you can draw your knowledge of the world. Then the question arises: what inferences are justified by this store of particulars and facts?[4]

It is the more restricted problem of the justification of the inference to material objects and to matter that Russell proceeds to discuss.

In the light of the statements of Russell's problem I have just given, I should like to return to the discussion of hard data. Previously I mentioned that hard data are the certain, or at least the most indubitable elements in knowledge, the elements that are left after all inference is excluded, and the elements that are to be used for the justification of the inferred elements. But the nature of the data that are to be used as a justification is unfortunately confused by considering a passage in *Our Knowledge of the External World* which I have not cited

[1] *Our Knowledge of the External World*, p. 73.
[2] *Ibid.*, p. 75.
[3] *The Analysis of Matter*, pp. 191–2.
[4] *Journal of Philosophy*, 1919, 21–2.

so far. Basic data, from the point of view that I have been presenting, would be those indubitable beliefs, regardless of their pyschological priority, which are the basis for the justification of inferred beliefs. These data would in fact often be arrived at only after considerable 'scrutiny' and elimination of inferences and 'interpretations'. However, in this further passage, basic data are considered from a psychological standpoint, as our psychologically primitive beliefs. Using data in this sense, Russell's problem would be to justify our inferred knowledge on the basis of our indubitable psychologically prior beliefs.

The emphasis in Russell's exposition in *Our Knowledge of the External World* is psychological, implicitly assuming that Russell's problem is such that it is relevant to investigate the origin of our beliefs, and the psychological aspect of the inferences to material objects. In addition, in that work there is an explicit statement that our derivative beliefs should ultimately be reduced to our psychologically primitive beliefs. I have already discussed the distinction between primitive and derivative knowledge, in *Our Knowledge of the External World* Russell makes a further distinction of derivative and primitive knowledge into psychologically and logically primitive and derivative beliefs.

> Here we become involved in a somewhat puzzling entanglement of logic and psychology. Psychologically, a belief may be called derivative whenever it is caused by one or more other beliefs, or by some fact of sense which is not simply what the belief asserts. Derivative beliefs in this sense constantly arise without any process of logical inference, merely by association of ideas or some equally extra-logical process. . . . In such a case, the knowledge is derivative psychologically; but logically it is in a sense primitive, since it is not the result of any logical deduction. . . . If we call a belief 'logically primitive' when it is not actually arrived at by a logical inference, then innumerable beliefs are logically primitive which psychologically are derivative.[1]

Not only, he maintains, should derivative beliefs be examined to see if they can be reduced to primitive beliefs, but logically

[1] *Our Knowledge of the External World*, p. 69.

primitive beliefs in turn should be reduced to psychologically primitive beliefs:

> When we reflect upon the beliefs which are logically but not psychologically primitive, we find that, unless they can on reflection be deduced by a logical process from beliefs which are also psychologically primitive, our confidence in their truth tends to diminish the more we think about them.[1]

Instead of using our 'critical scrutiny' to find the indubitable elements of our knowledge, which can then serve as a basis for all the rest, we need only find the psychologically prior elements, they will serve as the justification for the remainder of our knowledge. They will also be the indubitable elements, as Russell says:

> We do not feel this as regards the immediate objects of sense: there they are, and as far as their momentary existence is concerned, no further argument is required. There is accordingly more need of justifying our psychologically derivative beliefs than of justifying those that are primitive.[2]

The interpretation of Russell's problem suggested by such psychological considerations is not, I believe, the one with which he is really concerned. Not only do I feel that my view is supported by my previous discussion, but also by several explicit statements of Russell's. In the 1919 paper he distinguishes between various senses of the term 'primitive data', two of which are relevant here: (1) Pure psychology:

> What earlier beliefs preceded those which we now entertain, either in the individual or in the race? . . . And what vaguer objects than those presented to a trained observation are to be found in a less sophisticated experience? All these are questions of psychology. They are questions which I, for my part, have not attempted to discuss. Nothing that I have said on the problem of the external world is intended to be applicable to them.[3]

[1] *Our Knowledge of the External World*, pp. 69–70.
[2] *Ibid.*, p. 70.
[3] *Journal of Philosophy*, 1919, 8.

(2) Mixed psychology and logic (which he calls 'epistemological'), the sense in which Russell maintains he is concerned with data:

> What, if we are challenged, and an attempt is made to make us doubt the truth of physics, shall we fall back upon as giving a basis for our belief which we are not prepared to abandon? Take say, the facts out of which modern physics grew: Galileo's observations on falling bodies. We have in Galileo's work a mixture of argument, inference, mathematics, with something else which is not argued or inferred, but observed. For him, this something else constituted part of what was logically primitive. . . . We will call the primitive in this sense 'epistemological primitive'. It is the primitive in this sense that I mean when I speak of 'data'.[1]

And later in the same paper Russell discusses his problem from a point of view consistent with the quotations I have given in the preceding paragraphs.

In later works, this view is stated even more definitely.

> We said that it is the business of epistemology to arrange the propositions which constitute our knowledge in a certain logical order, in which the later propositions are accepted because of their logical relation to those that come before them. It is not necessary that the later propositions should be logically deducible from the earlier ones; what is necessary is that the earlier ones should supply whatever grounds exist for thinking it likely that the later ones are true. When we are considering empirical knowledge, the earliest propositions in the hierarchy, which give the grounds for all the others, are not deduced from other propositions, and yet are not mere arbitrary assumptions. They have grounds, though their grounds are not propositions, but observed occurrences. Such propositions, as observed above, I shall call 'basic' propositions.[2]

And Russell's problem in Part III of *Human Knowledge* is to examine the grounds for the inferences from data to the knowledge derived from that data. There is a change in the manner in which he states his problem from that found in *Our Knowledge of the External World.* In the latter book he asked whether anything *could* be inferred from data, the question in his latest

[1] *Journal of Philosophy*, 1919, 8–9.
[2] *An Inquiry into Meaning and Truth*, p. 21.

work is, since many things are obviously inferred from data, what principles of inference must be used in addition to data. But the essential point is the same. Russell does not now maintain that these inferences should not be made, even though they are not logically demonstrative, but he is concerned upon what principles of inference they are based.[1]

Without the psychological emphasis, Russell's problem, as I interpret it, is to find the 'certain' and non-inferential basis of our empirical knowledge, and to show how the remainder is founded upon that basis. But the discussion of psychologically prior data implies that the problem is one for which we need to conduct the psychological investigation into what elements actually come first in our obtaining knowledge. If that is the case, then the relation should be made clear between such a psychological investigation and determining the 'certain' basis of our knowledge. Of course, it is possible that the indubitable non-inferred base we are looking for is also that which is psychologically prior, but if so then the criteria of that base is its indubitability and non-inferred quality, and its psychological priority is only an incidental quality. If it is maintained by Russell that we should look for beliefs which are psychologically prior, without regard to 'certainty', then whatever problem is involved here is not philosophical but the psychological one of investigating the actual processes of knowledge. If it is maintained that Russell believes that we should look for psychologically prior beliefs because their priority gives them the certainty and non-inferred quality he desires, then this is a view which I believe Russell does not in general hold, or at least does not argue for at any length. I cannot, then, see the relevance of the psychological discussion in *Our Knowledge of the External World*; and I shall continue to assume that Russell does not intend to be concerned with a problem wholly or in part psychological. His lack of clarity in this respect is, however, unfortunate.

[1] *Human Knowledge*, pp. 165 ff., 175 ff.; for a similar view, see *The Analysis of Mind*, pp. 297 ff.

In *Human Knowledge* it is clear that psychological priority is irrelevant.

> Such beliefs I shall call 'data'. In ordinary thinking they are *causes* of other beliefs rather than *premises* from which other beliefs are inferred; but in a critical scrutiny of our beliefs as to matters of fact we must whenever possible translate the causal transitions of primitive thinking into logical transitions, and only accept the derived beliefs to the extent that the character of the transitions seems to justify.[1]

Distinguishing the element of 'interpretation' from the sensational core of a perception is, as I discussed earlier, necessary to determine what is really data, to find what premises are really given by the perception. Such interpretations, or inferences, may be 'spontaneous', psychologically as prior, if not more so, than the sensational core, but that spontaneity does not make them data.[2] In the analysis of our knowledge it is necessary to determine whether beliefs which seem to be spontaneous are as certain as their spontaneity might seem to make them, or if there is not a problem of justifying those beliefs by finding what premises they rest upon and what principles of inductive inference would allow us to arrive at those beliefs. A view similar to that of *Human Knowledge* can also be found in the passage I quoted from *An Inquiry into Meaning and Truth*.[3]

It would seem from these considerations that Russell does not really intend to be concerned with a psychological investigation of the origin and priority of beliefs, but in his discussions there is a further reference to psychology whose significance is not clear. In the 1919 paper[4] although he states that he is not concerned with psychologically prior data, the epistemological data with which he is concerned are characterized as mixed psychology and logic. In *An Inquiry into Meaning and Truth*, he says

> The first step in such a scrutiny is the arrangement of what we think we know in a certain order, in which what comes later is

[1] *Human Knowledge*, p. 166.
[2] *Ibid.*, pp. 166 ff., 182 ff.
[3] See above, p. 98.
[4] See above, p. 98.

known (if it is known) because of what comes earlier. This conception, however, is not so clear as it might seem to be. It is not identical with logical order, nor yet with order of discovery, though it has connections with both.[1]

A more detailed discussion of this point will be found later in *An Inquiry into Meaning and Truth*, Chapters IX, X. He discusses the notion of 'epistemological premises' and includes among their characteristics that (*a*) they are 'psychological premises'. 'A psychological premise may be defined as a belief which is not caused by any other belief or beliefs.'[2] It is what Russell called in *Our Knowledge of the External World* a 'psychologically primitive' belief, one that results directly from perception. Yet psychological premises cannot be accepted as epistemological premises unless they possess the further characteristic (*b*) being true as far as we can ascertain. Only those should be accepted of which we are most certain, those that can be doubted would not truly be epistemological premises. A sub-class of epistemological premises Russell calls 'basic propositions', which 'are caused, as immediately as possible, by perceptive experiences'.[3] These propositions constitute, in *An Inquiry into Meaning and Truth*, what Russell elsewhere has characterized as hard data of perception. Corresponding to epistemological premises, basic propositions must possess the two characteristics that (*a*) they arise on the occasion of some sensible occurrence, and (*b*) they are of such a form that they will not be contradicted by other basic propositions. It is (*b*) which is the limiting characteristic, for there may be many propositions which conform to (*a*) but not to (*b*). Beliefs that are 'caused' or arise from perceptive experience, are as we have seen, often not data; for if they contain an element of inference, as, for example, 'there is a dog', they can be contradicted, since they go beyond the present data and may be refuted by subsequent evidence.[4] In determining what is really a basic

[1] *An Inquiry into Meaning and Truth*, p. 15.
[2] *Ibid.*, p. 165.
[3] *Ibid.*, p. 171.
[4] *Ibid.*, p. 173.

proposition it is necessary to analyse the proposition in question and discard as many inferential elements as possible.

Searching for propositions that cannot be contradicted, or that are as certain as we can ascertain, is not the psychological enterprise suggested by *Our Knowledge of the External World.* The relevance of psychology to data is that it is psychological evidence from perception which provides the evidence for basic propositions (or which Russell sometimes says *is* itself data); it is perception which is the cause of basic propositions, and at the same time the evidence for them.[1] But it is not every perception which can be simply used for evidence, and not every proposition caused by perception that is a basic proposition. Only those propositions are accepted as basic propositions which meet the standards of analysis set down in condition (*b*). These standards are not psychological standards, and since it is this analysis which is of primary importance in determining what is hard data, it is safe to conclude that the analysis of knowledge which Russell is performing is not one that depends on psychological criteria.

So far I have been concerned primarily with presenting Russell's views, making only few attempts at clarifying or criticizing them. However, it would be useful now to examine several important terms in Russell's discussion and by that means obtain a clearer view of the problem with which he is apparently concerned. For the present I shall ignore Russell's metaphysical views as much as possible, confining myself to his epistemological analysis, even though in his own presentation there is often no clear separation between epistemological and metaphysical beliefs. It will be a further problem to determine the extent to which these two groups of beliefs are independent of each other.

It is important in evaluating empirical knowledge, Russell believes, to distinguish the 'certain' elements of that empirical knowledge, upon which the remainder should be based. One might very well wonder what Russell means by 'certain' know-

[1] *An Inquiry into Meaning and Truth,* pp. 172, 174.

ledge. In Russell's own writings there is a shift in the emphasis which he places upon 'certainty' as a criterion of that basic knowledge. Thus in *The Problems of Philosophy* 'certainty' is the primary characteristic of basic knowledge, in *Our Knowledge of the External World* 'certainty' is coupled with 'non-inferred', in other writings we find 'certainty' absent and 'non-inferred' the predominant characteristic, and in *An Inquiry into Meaning and Truth* and *Human Knowledge* data are 'non-inferred' and possibly not 'certain'.[1] There are several senses of 'certainty' with which Russell is not concerned. One is the use of 'certain' when it is applied to propositions reached by processes of valid deductive inference. This is hardly the meaning Russell intends here, since he is primarily concerned with initial propositions, ones which are specifically not arrived at by any such process of inference. However, could the certainty characteristic of propositions arrived at by deduction in some way be ascribed to the premises? One sense in which this could be true I shall discuss below. Another meaning this could have is that the feeling of certainty we have about the conclusion is the same kind of feeling that Russell believes we could have about the premises when they are hard data. To what extent is Russell concerned with subjective feelings of certainty in distinguishing between certain and uncertain beliefs?

Often Russell does refer to beliefs that one *feels* to be certain, as if the individual's feelings of certainty were the important criterion in distinguishing such beliefs from others. But rather than being a feeling that is accepted uncritically, it is one that follows upon certain definite logical considerations. If the uncritically accepted feeling of certainty were to be a guide, then anyone's certainty would be as good as the next man's. Everyone could then proceed to arrange his beliefs beginning with those he felt to be most certain and going on to those least certain. On this basis, however, since those not used to philosophical analysis are in the great majority, we would find that the large majority of people would find the most certain beliefs

[1] *An Inquiry into Meaning and Truth*, p. 155; *Human Knowledge*, p. 391.

to be those of common sense, those beliefs which Russell believes to be not at all certain. But those accustomed to philosophical analysis have come to doubt ordinary beliefs only after considerable thought and effort, and they can produce definite grounds for justifying their doubt of such ordinary beliefs. The feeling of certainty may very well accompany the beliefs both of the philosopher and those not accustomed to philosophy; it is hardly a trustworthy guide in distinguishing basic from derived beliefs, but rather an accompanying satisfactory state of mind when the final results have been obtained. The feeling itself is not a standard which Russell employs in distinguishing hard and soft data.

The grounds then upon which one comes to doubt beliefs customarily considered certain are the characteristics we should examine to find what to Russell constitutes the 'certainty' of a belief. The essential characteristics of these 'certain' beliefs is that they are 'non-inferred'. To determine this certain part of our knowledge it is necessary to examine our beliefs to find whether or not they contain elements of inference. We might very well also *feel* certain of these non-inferred beliefs, but our feelings are not the important consideration. Knowledge known without inference is further characterized by Russell as 'self-evident', where 'self-evident' means 'believed on its own account', or as he says later, 'credible independently of any argument in their favour'.[1] I believe that the distinction which Russell makes between certain and uncertain knowledge is not on the grounds of whether or not inference is *in fact* present, but whether or not the evidence necessary to establish the belief can be completely obtained. A certain belief is one which should 'be believed on its own account', because no further evidence than that already present is required to maintain it. Thus if a brown sense-datum is present to an individual, and he believes 'what looks like a brown patch is occurring now' (overlooking crudities in the verbal expression), the brown sense-datum is all the evidence required to establish his belief.

[1] *An Inquiry into Meaning and Truth*, p. 17.

However, if he were to say, 'I see a brown cow' further tests and observations would be required to show that this was in fact the case, that what he was seeing was a cow rather than an illusion. When the individual made this latter statement, his belief could not be classed as certain in any high degree, since none of these further observations had been made. Ordinarily, as he would make further observations the belief would become more certain, but it would remain an inference, since it would depend on the truth of various statements reporting observations. Inferred beliefs are uncertain to a varying degree, because their truth depends upon many pieces of evidence, and future evidence theoretically may show the belief to be unsound.

This interpretation of 'certainty' is more explicitly stated in *An Inquiry into Meaning and Truth* and *Human Knowledge* than in earlier works.[1] 'Data' are defined as propositions which are 'believed on their own account', or having 'some degree of rational credibility on [their] own account, independently of any argument derived from other propositions'.[2] Data given in perception obtain their normally high degree of credibility (usually practically 'certainty') by the evidence only of the sensation present. There may in addition be evidence for them derived from other propositions, but this does not affect their certainty as data. Propositions derived from data, but not themselves data, require arguments derived from other propositions to substantiate them. They are inductive inferences, not demonstrative inferences, and hence are less certain than data since the evidence for their truth is never all present. They may be contradicted on the basis of additional evidence, but data cannot be since there can be no additional evidence for them. The use of 'certainty' here is similar to the sense in which one

[1] *An Inquiry into Meaning and Truth*, p. 156; *Human Knowledge*, pp. 391 ff. R. M. Chisholm's 'The Foundations of Empirical Knowledge', in *The Philosophy of Bertrand Russell* (pp. 421 ff.) discusses Russell's notion of data, and arrives at a conclusion which I believe supports my interpretation of Russell's views.

[2] *Human Knowledge*, p. 392.

might speak of a valid deductive inference being certain since no evidence other than that stated in the premises is necessary to establish the conclusion, and since with true premises the conclusion cannot be false. In the case of data, there is no argument involved, but the one proposition in question requires only the evidence provided by the sensation to establish its truth.

The difficulty with using the word 'certainty' is that it often carries with it connotations of the subjective *feeling* of certainty, which as I have pointed out, is not I believe what Russell intends. Russell himself has not clarified matters any by frequently speaking of 'certain' beliefs as beliefs that are 'indubitable' or those that we cannot doubt, where the emphasis seems to be placed on the actual psychological act of doubting. But here again I believe that the 'doubt' is essentially one based on logical conditions. Beliefs that cannot be 'doubted' are those that are certain in the sense I have discussed. In *Human Knowledge*, Russell introduces the word 'credible' to refer to the objective grounds of belief, while 'certain' refers to the individual's feelings about those beliefs.

> I think, therefore, that everything we feel inclined to believe has a . . . 'degree of credibility'. . . . It is not, however, purely subjective. There is a cognate subjective conception—namely, the degree of conviction that a man feels about any of his beliefs—but 'credibility', as I mean it, is objective in the sense that it is the degree of credence that a *rational* man will give.[1]

There is not necessarily a close correlation between credible beliefs and an individual's feelings of certainty concerning them. In many cases, people are 'certain' about beliefs which logically are wholly unwarranted, while on the other hand, a 'rational' individual will have his feelings of certainty or uncertainty more closely connected with the objective credibility of his beliefs.[2] It is the objective grounds of subjective certainty, or credibility, with which Russell is concerned. It is unfortunate

[1] *Human Knowledge*, pp. 342–3.
[2] *Ibid.*, pp. 396 ff.

that this distinction had not been more explicitly applied in earlier works, or even in other sections of *Human Knowledge*.

The use of 'inference' in the preceding paragraphs suggests that Russell is concerned with a meaning of the term not as broad as that which he actually frequently gives it. His discussion of inference is not as clear as one might wish. Some inferences are explicit and conscious, as are those in most conscious reasoning; recognizing them and finding their relative certainty or uncertainty offers no great problem. The difficulty with the use of 'inference' comes with applying the term to those cases that are not conscious. Russell himself is aware of this difficulty, as we recall from his discussion of the vagueness of the boundary between hard and soft data. If we interpret 'physiological inference' as applying to a definite act taking place, as Russell apparently intends that we should, then it hardly seems to be inference in any customary sense, but a sequence of physical cause and effect. Many of the beliefs which Russell maintains are physiological inferences arise 'spontaneously', and can be classed as inferences only by finding psychological causes for such beliefs. Granting, however, that such beliefs are legitimate inferences, and consequently not hard data, we encounter further difficulties with another statement of Russell's. He says that that part of a belief which can only be shown to be inference by an elaborate theory should be regarded as part of the datum. This statement seems to apply to such 'spontaneous' inferences, since the psychological causes are found only as the result of a psychological analysis.[1] In later works, however, he apparently returns to the view that all elements of inference should be eliminated as far as possible.

The discussion of inference has, I believe, been unnecessarily confused because Russell has, as he did also in the discussion of hard data, introduced psychological considerations which are not relevant to the problem with which he is really concerned. In the discussion of physiological inference in

[1] *The Analysis of Matter*, p. 189.

Philosophy Russell has attempted to justify the application of 'inference' to psychological processes by showing that the process in question can be analysed into psychological cause and effect. Similarly in *Our Knowledge of the External World* a similar process is referred to as 'instinctive inference' in which we associate images and expectations with a sensory stimulus to give us a perception.[1] Such discussions purport to describe a process which actually takes place in an individual's perceptions, attempting to show that various beliefs are *in fact* the product of certain psychological processes. Calling these processes 'inference', the beliefs arrived at by this means will be inferred beliefs. If, however, we follow Russell's statements of his problem, and the interpretation of that problem I have been suggesting, such psychological investigations are irrelevant, since the criterion of a derived belief is not whether it is *in fact* immediate or derived, but is found in the logical grounds for accepting such a belief.

Beliefs that are not inferred are certain beliefs and are those which as I have mentioned can be established entirely by the evidence present with the belief. Beliefs concerning the presence of sense-data to an individual require for their truth only that those sense-data be present, they are 'believed on their own account' since no additional evidence would be relevant. When we examine derived, or inferred, beliefs we find that they are beliefs requiring more than the sense-data present to the observer for their proof—we 'infer' that such beliefs, e.g. those concerning material objects, are true on the basis of the sense-data present to us, of sense-data obtained as a result of certain actions on our part, of sense-data reported by other persons. Similar to other beliefs arrived at by an inductive inference, these beliefs are 'uncertain' because the complete evidence theoretically required for their justification cannot be obtained.

One might maintain that physiological inferences are inferences in the sense I have just used the term, since we can by

[1] *Philosophy*, pp. 13, 80 ff.; *Our Knowledge of the External World*, p. 68.

attention become conscious of many of them.[1] Thus the belief in material objects is spontaneous, but upon critical reflection it might be said that we can become conscious of believing first in certain percepts, then inferring upon the basis of these the belief in the existence of the material object. However, to me it seems that what has been done here is not to become conscious of a psychological or physiological process, but to analyse the belief into certain and uncertain elements, performing, in short, the epistemological analysis that I am maintaining is the one in which Russell is primarily interested. Thus those beliefs that are 'inferred' in the sense important here, although including many beliefs which are in fact inferred, are beliefs whose objective status requires some justification. 'Data' are beliefs the evidence for which is all present, while 'inferred' beliefs are those which do not meet this test. Whether they arise spontaneously is not the important consideration, but given those beliefs, can they be established as data, or only on inductive grounds? In the latter case, they are 'inferred' beliefs.

Following the interpretation I have given to Russell's use of 'certain' and 'inference', Russell's problem can be restated as in part being the critical examination of common-sense beliefs to find the most certain elements in those beliefs, and, using these as premises, to show the inferences that justify the belief in question. The common-sense belief which is Russell's special problem is that of the existence of material objects. Our most certain beliefs, Russell maintains, are those in our own percepts. The belief in material objects is founded on correlations between percepts of one individual, percepts of other persons, and percepts which are anticipated and actually obtained as the result of various actions; it is an uncertain belief because it is an inductive inference resting on evidence of the existence of such correlations, evidence which cannot be complete, since the possible correlations are inexhaustible. The manner in which Russell interprets this belief in material objects will be shown in the following Chapter. The task of

[1] *The Analysis of Matter*, p. 189.

'justifying' such derived beliefs is not necessarily to show that they are certain, but to examine the grounds we do have for believing them, and find the degree of certainty which the beliefs seem to warrant. If we could apply this procedure to all our common-sense beliefs, we could show what is certain and uncertain in our empirical knowledge, the logical base upon which it rests, and the degree of certainty which the remainder possesses.

The direction in which we should look for hard data seems clear, but Russell has had considerable difficulty in determining just what is hard data; it will be remembered that he can find no precise dividing line between hard and soft data. Hard data include our own 'sensations' or 'sensible objects' or later, 'percepts', or what are frequently called 'sense-data'. Russell has found it difficult to separate percepts from elements of interpretation and inference, and has said that they are 'limiting conceptions' to which we approach 'asymptotically'.[1] The simplest way to determine what is a percept would seem to be by applying the criterion of certainty, when certainty means non-inferred in the sense of all the evidence required to establish the belief being present—the percept being that evidence. Russell's discussion of hard data is unfortunately confused, principally I believe because of the introduction of irrelevant psychological factors I have previously mentioned. At one place in *Our Knowledge of the External World*, Russell states that those elements which are psychologically prior in our perception are also the hardest of data. In the discussion of physiological inference, and also, e.g. *The Analysis of Matter*,[2] Russell attempts to show that many beliefs are founded on psychological association and consequently are inferred. I have already stated why I think the introduction of psychological elements is aside from the point Russell is examining. Russell's discussion implies some definite criteria of hard data in terms of certainty and lack of inference, and if psychological data are to be ad-

[1] *An Inquiry into Meaning and Truth*, p. 155.
[2] P. 182.

nitted it would have to be made clear that they meet such a est of certainty, or that there are other grounds upon which hey should be admitted. On the other hand, the criticism gainst Russell on the ground that he holds an 'atomistic' sychology would not be relevant to his epistemological prob- em, unless his psychological beliefs are supposed to support is philosophical theories. But for his epistemological problem t is not that he holds an unsound psychology that is a valid riticism, but that he introduces one at all. The discussion of pre-analytic' vs. 'post-analytic' data is also not relevant, since his discussion concerns the temporal priority of data, and not he fact that they are certain and indubitable.

A point upon which a more precise discussion by Russell vould be desirable is his distinction between percepts and eliefs (or propositions) about them. In the earlier writings I ave been discussing it is not clear whether hard data include ercepts, beliefs about percepts, or both. Precepts, or sensible bjects, are simply occurrences—like events, they just *are*; ut beliefs are characterized by being the kind of thing that an be true or false. Are these occurrences among our 'certain' lata, or is it only beliefs about them that can be? If it is these eliefs, for which presumably the occurrences are evidence, hen the relation between the occurrences and the beliefs about hem must be determined. We could interpret Russell to mean hat only a *belief* could be certain, consequently percepts vould not themselves be hard data, but would be the evidence hat would completely verify beliefs about percepts, thus naking them hard data. On the other hand, Russell could be nterpreted as meaning that percepts themselves are certain, ince nothing could be more 'certain' than an actual occur- ence. In this case, further discussion would be necessary to letermine the degree of certainty of beliefs about percepts. This latter interpretation is one which is opposed by those who naintain that the mere occurrence of percepts cannot constitute knowledge', but that something of the nature of belief is equired for knowledge to be present. Russell at one time

dismisses this point as a verbal argument, in effect saying that he can consider sense-data knowledge if he wishes. However, the general distinction between percepts and beliefs about them still remains, and is one to which Russell usually gives only the briefest attention.[1] In fact, in *Philosophy* we find the following confusing discussion:

> The actual datum, in each case, is unimpeachable, but the extensions which we make instinctively are questionable. . . . No words exist for describing the actual occurrence in all its particularity; all words, even proper names, are general, with the possible exception of 'this', which is ambiguous . . . what is really a datum is unutterable, and what can be put into words involves inferences which may be mistaken (12–13).

However, later we find the following definition:

> A 'datum' is a form of words which a man utters as the result of a stimulus. . . . If the above definition is accepted, all our data for knowledge of the external world must be of the nature of percepts (266–7).

If Russell believes these above statements do not conflict, it would be desirable to show why they do not, or at least that the apparent conflict is unimportant.

In *An Inquiry into Meaning and Truth*, however, Russell does recognize the distinction between percepts (or sensations) and propositions concerning them, and discusses at length the relation between them.[2] Data of perception are propositions, the evidence for which consists of some perceptual experience. Unlike his earlier view, Russell states that the perceptual experience itself has no cognitive value, and that premises for empirical knowledge are not such experiences, but propositions based in some sense upon the experiences.[3] But in *Human Knowledge* the distinction is not nearly so clear. Thus we find that 'percepts' are data,[4] 'sensations' are data,[5] 'beliefs' are

[1] See above, pp. 89–90.
[2] *An Inquiry into Meaning and Truth*, Chapters, III, VII, VIII, IX, X, XI.
[3] *Ibid.*, p. 149.
[4] *Human Knowledge*, Part III, Chapter IV.
[5] *Ibid.*, p. 170.

data,[1] and 'propositions' are data.[2] The relation between 'percepts' and 'sensations' I have already discussed.[3] The distinction between 'beliefs' about percepts and 'propositions' concerning percepts can also, I believe, be overlooked, if we assume that the former emphasizes psychological elements, while the latter is more concerned with the logical elements. For the purpose of Russell's discussion the two terms are closely enough related so that the difference between the two would not be significant.

A more serious problem is whether data of perception are perceptual events or propositions. The proper selection of quotations from Russell's writings will support either view, but if we give his latest works most weight, I believe we will find the balance in favour of data being propositions. Fortunately, it is unnecessary to attempt to solve this problem, because I believe that Russell's general problem would be essentially the same whatever view was adopted. If, however, we adopt the interpretation suggested by *An Inquiry into Meaning and Truth*, which I find most plausible, it is propositions that are certain, or uncertain, and that must be justified. The actual percept is a physical occurrence and is the evidence for the proposition. As an occurrence it cannot be 'certain'.

So far in this Chapter I have been presenting a problem, primarily one of epistemology. Our empirical knowledge has degrees of certainty and uncertainty, corresponding to the extent of inference involved. Russell has discussed the character of that knowledge which is most certain, or which contains as few elements of inference as possible, but has not as yet discussed the justification of the inferences to the less certain portions. His discussion concerns specifically the character of scientific knowledge, and the common-sense belief in permanent material objects. I have not questioned the general distinction which Russell is making between certain and uncertain knowledge, but I believe that granting this distinction, it is

[1] *Human Knowledge*, p. 166. [2] *Ibid.*, p. 392.
[3] See above, p. 88.

possible to provide an alternative analysis of knowledge that will conceive of the 'certain' elements in knowledge in a manner which will avoid some of the difficulties in Russell's analysis. I shall suggest such an analysis in the concluding Chapter.

In many of Russell's writings, especially his earlier ones, although we find him attacking the epistemological problem I have outlined, there is in addition an introduction of metaphysical considerations so that his problem becomes more complex. Thus 'Can we know that other *objects*, inferable from *objects* of sense but not necessarily resembling them, *exist* either when we are perceiving the objects of sense or at any other time?'[1] Not only do we have an epistemological problem, but now at the same time a metaphysical one. I shall discuss briefly Russell's metaphysical views, primarily in his earlier works, and their relation to the epistemological problem I have been discussing so far.

It is important to present Russell's metaphysical views on the nature of 'percepts', 'sensible objects' or 'sense-data' although it is not necessary to discuss his general metaphysical position at this time. In my own discussion I have tried to use those terms in as neutral a manner as possible, but Russell does not intend to do so and his metaphysical views affect the problem he believes he is dealing with. Around 1914, the metaphysics which Russell maintained was a realism exemplified in some respects by Samuel Alexander and T. Percy Nunn, to whom Russell gives credit for some of his ideas.[2, 3] The position is an attempt to avoid an idealism like Berkeley's, to keep the distinction between the mental and the physical, but without the separation of primary and secondary qualities which Locke had made. Russell distinguishes between '(1) our sensation, which is a mental event consisting in our being

[1] *Our Knowledge of the External World*, p. 75 (my italics). See above, p. 95.

[2] *Mysticism and Logic*, p. 125. See the paper by Nunn in *Proc. Arist. Soc.*, N. S. x (1909–10).

[3] A summary of Russell's metaphysical development is in Morris Weitz's essay in *The Philosophy of Bertrand Russell*, pp. 58 ff.

aware of a sensible object, and (2) the sensible object of which we are aware in sensation.'[1] The sensation is a mental event, where mental events include events such as desiring, willing; the sensible object, or sense-datum, is the object of this mental event, and is physical. Russell, like others at this time, did not want to follow Berkeley and other idealists in making sense-data mental; he maintains that they are physical, while it is the awareness of them that is mental. But though sense-data are physical, they are still subjective, since they depend in part on the physiological nature of the observer; they are not permanent material objects common to many, but are private to the observer. Russell locates them in the brain. It is sense-data, particular, physical, yet non-persistent, dependent upon our bodies, that Russell calls the 'ultimate constituents of matter'.[2] Later, Russell abandons the distinction between the sensation and the sensible object. Sensible objects, or sense-data, continue to have the properties I have just described, but the notion of 'mental awareness' of them has been discarded. I will discuss later what then becomes of 'mind'.[3]

Sense-data, or sensible objects, or what Russell later called 'percepts', are what we observe in perception. In *The Analysis of Matter* and *Human Knowledge* 'percepts' are a sub-class of 'events'. We do not observe material objects, but observe instead percepts from which we infer in some way the existence of material objects.[4] It is a theory similar to earlier British empiricism, by which, of course, Russell was influenced.

[1] *Our Knowledge of the External World*, first edition, p. 76. This distinction is made only in the first edition of this work (1914). It is abandoned in the second edition (1929) where Russell's views on the nature of sensations conforms more to that of *The Analysis of Mind* (e.g. pp. 141 ff.) and later works. There are few changes between the two editions of *Our Knowledge of the External World* other than on this point.

[2] *Mysticism and Logic*, pp. 125 ff.; *Our Knowledge of the External World*, first edition, p. 76.

[3] *Our Knowledge of the External World*, second edition, p. 80: 'When I speak of sensation *or* sensible object . . .' (my italics). *Analysis of Mind*, pp. 141 ff.

[4] I am disregarding the question whether it would be more correct to say that the percepts are our observations, rather than objects which are observed.

Determining precisely what is any one percept is not a simple matter, it is subject to the same difficulties we met in trying to determine what was hard and what was soft data.

A well-known example of Russell's of what, according to this view, is the correct interpretation of observations purporting to be of the physical world is the following:[1] When a physiologist is observing the brain of another person, ordinarily one speaks of what he is observing as being in the brain of that other person. However, according to Russell, what every person perceives are really percepts, located somewhere in his own brain. Thus the physiologist is observing percepts which are located within his own brain, and by some process of inference, relating them to the brain of the person he is observing. Thus we arrive at the startling result that when the physiologist observes the brain of another, he is really observing something within his own brain. This example has caused considerable surprised comment, since it is a strange result judged by more common-sense theories. But within Russell's theory it is an intelligible analysis,[2] since according to that theory we observe percepts, located within our brains, and the further relation of percepts to material objects is obtained by inference.[3]

I have attempted to keep Russell's epistemological and metaphysical views separate in my presentation, since I believe that they are essentially distinct. Actually the two views are combined in Russell's discussion, so that the problem I have discussed as an epistemological one is not clearly stated as such, but as a joint epistemological and metaphysical problem. The result is a confusion which if not clarified will make very difficult any sound evaluation of Russell's analysis. Admitting Russell's epistemological analysis of knowledge into certain and relatively uncertain elements, with the most certain being our own percepts, it is not, I believe, necessary to accept his meta-

[1] *Philosophy*, pp. 140 ff.; *Human Knowledge*, p. 229.

[2] See Daniel Cory, 'Are Sense-data "in" the Brain?' *Journal of Philosophy*, XLV (1948), 533–48.

[3] See below, pp. 151, 174.

physical views as to the nature of those percepts. Russell has found that our certain beliefs are like 'brown patch occurring now'. It does not follow from this statement that 'brown patch' refers to a particular existent located in our heads. If this latter statement is true, it must be on other grounds than the truth of the statement reporting the sensible experience. The epistemological and metaphysical views are not inconsistent, but neither does his epistemological analysis require one to accept his metaphysics. Accepting his belief that the most certain knowledge is that of our own percepts, there are several views as to the status of those percepts that one could hold. Percepts might be shown to be mental, as in Berkeley, they might be physical, as in Russell's own view, or they might be the external object itself as we see it from a particular point of view. Whichever of these views, or other views, one would accept, depends on arguments based on further beliefs than those derived only from the epistemological analysis.

His epistemological distinctions are not, I maintain, arguments for accepting the view of the nature of percepts which Russell adopts. The epistemological analysis assumes that view, but could be performed without any metaphysical assumptions equally well, as is done in many passages in *An Inquiry into Meaning and Truth* and *Human Knowledge*. Russell does not, I believe, maintain that his analysis does give a basis for accepting his metaphysics, although the contrary sometimes may appear to be the case. However, assuming his metaphysical views as to the nature of percepts, and his epistemological analysis, Russell, I believe, does maintain that metaphysical views according to which we have knowledge of a permanent objective substance, 'material object', are questionable.

PART II

LOGICAL ATOMISM

The general philosophical problems that are discussed in Part I and Part III of this Chapter are essentially the same

problems as were raised from another point of view by the analysis of logic, of language and meaning, which Russell maintained during his 'logical atomism' period. These logical doctrines can be found in such works as *Our Knowledge of the External World* (1914), Russell's Introduction to Wittgenstein's *Tractatus Logico-Philosophicus* (1922), *The Analysis of Mind* (1921), to some extent, and especially his 'Philosophy of Logical Atomism' lectures, 1918–19. It is a conception of logic which finds a close similarity between the structure of language and the structure of the world, a conception which Russell has abandoned in the comparatively simplified form in which it is found during the writings of this period. His views in *An Inquiry into Meaning and Truth* and *Human Knowledge* on the points discussed here are more complex; the principal philosophical problems are stated in the logical atomism period in terms which I believe he would no longer accept.

In this Part, I shall discuss his views during this period in sufficient detail to make clear their bearing on the principal problems he raises, which, I believe, are in important respects the same as those I presented in Part I. In the preceding Part, I found it clearest to present his epistemology separately from his metaphysics, but it is pointless to attempt such a separation here. Although the reader may feel that the logical and philosophical doctrines at this time were inadequate, they should not be too hastily condemned, since they represent an important step in the development of contemporary philosophy. Since they are views which Russell has abandoned and are not, to my knowledge, advanced by contemporary philosophers, I have not criticized these logical doctrines, although I would not on the whole accept them.[1]

[1] A brief discussion of Russell's logical views at this period can be found in *Our Knowledge of the External World*, Chapter II. For a general statement of the position, see Russell's Introduction to Wittgenstein's *Tractatus Logico-Philosophicus*, London and New York, 1922, and his 'Logical Atomism', in *Contemporary British Philosophy*, First Series, London and New York, 1924. Further discussion will be found concerning 'belief' and 'truth' in *The Analysis of Mind*, Chapters 12 and 13. The most detailed exposition, which

His philosophy of logic Russell calls 'logical atomism'; the 'atomism' because Russell believed the world was composed of many separate things, the 'logical' because 'the atoms that I wish to arrive at as the sort of last residue in analysis are logical atoms and not physical atoms'.[1] His discussion of logic examines primarily the relation between language and the objective states of affairs to which language refers. The basic unit of discourse is the proposition, and consequently the discussion concerns especially the objective reference of propositions and their analysis into constituents. The reader need not be surprised at the similarities between Russell's views and Wittgenstein's,[2] since it was around this time that Russell was especially influenced by Wittgenstein.[3]

The basic unit of discourse, the proposition, is a complex symbol composed of parts which are also symbols.[4] A proposition, according to Russell's interpretation, is similar to what is often meant by 'sentence', a form of words with no further ontological significance. Russell's propositions do,

my discussion follows, is the 'Philosophy of Logical Atomism', *Monist*, XXVIII (1918), 495–527, and *Monist*, XXIX (1919), 32–63, 190–222, 345–380.

In *The Philosophy of Bertrand Russell*, pp. 81 ff., Morris Weitz also discusses these views briefly.

[1] *Monist*, 1918, 497.

[2] See Russell's Introduction to Wittgenstein's *Tractatus Logico-Philosophicus*, and *Monist*, 1918, 495.

[3] I am limiting my discussion to propositions and their objective reference, and not investigating the relation of propositions to beliefs, judgments, and assertions. This latter question is not, I feel, clearly explained by Russell at this time, but for my purposes it is fortunately not necessary to discuss the point. (See *Monist*, 1918, 504, 507–8; 1919, 56 ff. For a different view see *Our Knowledge of the External World*, p. 52. The fullest discussion is in *The Analysis of Mind*, Chapter 13.) At least it seems clear that propositions are essential elements in beliefs, judgments, and assertions, and that if we know the conditions for the truth of propositions we will know the objective conditions for the truth of our beliefs and judgments. Thus the objective conditions for the truth of our beliefs are the same as those for the truth of the propositions which state the contents of those beliefs.

[4] *Monist*, 1918, 504; 1919, 57; *Our Knowledge of the External World*, p. 52; *The Analysis of Mind*, p. 241.

apparently, have no metaphysical significance.[1] They are characterized by the property of being either true or false. The objective reference of propositions is what Russell calls 'facts'; if there are facts corresponding to propositions they are true, if not, they are false. The word 'fact' is used here in a technical sense, derived from, but not identical with, the ordinary use. The 'facts' with which true propositions correspond are not actually physically existing states of affairs, but subsistent entities, which have actual states of affairs as components. The entities out of which the world is built, Russell believed at this time, included 'ultimate simples' (of which percepts or sensible objects are one kind) and facts:

> . . . that those simples have a kind of reality not belonging to anything else. . . . There are particulars and qualities and relations of various orders, a whole hierarchy of different sorts of simples, but all of them, if we were right, have in their various ways some kind of reality that does not belong to anything else. The only other sort of object you come across in the world is what we call *facts*, and facts are the sort of things that are asserted or denied by propositions, and are not properly entities at all in the same sense in which their constituents are.[2]

Russell does not define his technical usage of 'fact', but limits himself merely to explaining what it is; a fact is that (whatever its exact nature) which makes a proposition true or false, the objective reference of every true proposition.[3] Facts themselves are not true or false, they just *are*; they are objective, independent of our thought, are not particular existing things that can be pointed at and given proper names.[4] Thus, if the proposition 'the sun is shining' is true, it is true because it corresponds to the fact that the sun is shining; but this fact is not the physical events which comprise the shining of the sun (assuming it *is* shining), but a metaphysical entity of which the

[1] *Monist*, 1919, 56.
[2] *Ibid.*, p. 365.
[3] *Monist*, 1918, 500; *Our Knowledge of the External World*, p. 52; *The Analysis of Mind*, p. 232.
[4] *Monist*, 1918, 501; *Our Knowledge of the External World*, p. 51.

physical sun is a constituent. It may be difficult to see from this example that a fact is different from physical events, but this will be more clearly seen in an example which I shall give shortly of a relational fact, and also from the presence of what Russell calls 'general' facts, those corresponding to true general propositions. General propositions, or at least those beginning with 'all', do not correspond with some physical state of affairs, but do, Russell maintains, correspond with some fact, which consequently cannot be identical with a physical state of affairs.

Propositions and facts are similar in structure; both are complex, composed of constituents, and, ideally, such that a one-to-one correspondence can be established between the constituents of each. Thus, if a proposition is composed of a relation word, and words standing for the two terms of that relation, the corresponding fact is composed of the relation itself and the two terms. Since both propositions and facts are complex, they may be analysed into the parts which compose them.[1] Ideally, to Russell, language would be such that in all cases its words (with the exception of logical constants) would have a one-to-one correspondence with the constituents of the facts expressed by propositions. In actual practice, this complete correspondence can only be approached; if it were actually attained we would have a logically perfect language. In the examples I shall discuss I shall assume this correspondence has been reached. It is the complexity of facts, accompanied by a corresponding complexity of language, which leads Russell to say 'there is an objective complexity in the world, and . . . it is mirrored by the complexity of propositions'.[2]

The nature of facts and propositions is more clearly shown by presenting in part Russell's classifications of facts and propositions. The simplest type are atomic propositions and facts. The constituents of atomic facts are terms and relations (or qualities).[3] Atomic facts form a 'hierarchy', the simplest of

[1] *Monist*, 1918, 514.
[2] *Ibid.*, 520.
[3] *Ibid.*, 522; *Our Knowledge of the External World*, p. 51.

which is a fact composed of one thing ('term') and a quality, the second, two terms and a diadic relation, the third, three terms and a triadic relation, etc. The propositions expressing these facts form a corresponding hierarchy and are called 'atomic' propositions. The simplest atomic proposition, that composed of words standing for one term and a quality, is the conventional subject-predicate proposition, in which the subject of the proposition refers to the term of the fact, and the predicate refers to the 'quality' or 'property' that the term has. Thus the atomic fact expressed by this type of proposition is composed of a term and a quality (or a term and a relation, if one considers, as Russell does, that a quality is a one-term relation).[1] An example of a higher level atomic proposition is one about the three-term relation 'between'. The atomic proposition 'this table is between the chair and the wall' corresponds to the atomic fact whose terms are 'this table', 'the chair', and 'the wall', and whose remaining constituent is the relation 'between'.[2] This last example shows that the metaphysical status of facts is not the same as that of physical events. The fact corresponding to 'this table is between the chair and the wall' is not simply the physical juxtaposition of the three objects in the order in which the words appear in the proposition. If that were the case, the real order of the objects could only be indicated by a corresponding order of the words in the proposition; this kind of correspondence is possible, and even then awkward, for only a few relations. The proposition, however, contains the word 'between', and this word corresponds to the *relation* 'between', which is an additional constituent of the fact besides the three objects.

Many types of propositions and facts are more complex than

[1] *Monist*, 1918, 522.

[2] *Ibid.*, p. 522; *The Analysis of Mind*, p. 278. Strictly speaking, this example is not correct. It does give a correct view of the correspondence between facts and propositions, but I have used 'term' in a wider sense than is really justified. In some cases, such as the preceding reference to *The Analysis of Mind*, when he is not concerned with a precise discussion of the nature of terms, Russell himself gives examples similar to mine.

atomic ones. 'Molecular' propositions are built up from atomic propositions by joining several of the latter by logical connectives such as 'or', 'if-then', or 'and'.[1] Molecular propositions are truth-functions of the component propositions; the truth or falsity of the whole proposition depends only on the truth or falsity of the component propositions. There are no molecular facts, Russell believes, corresponding to molecular propositions, rather each component atomic proposition refers to its corresponding fact.[2] Another type of proposition is that containing verbs such as 'believe', 'wish', 'desire', which Russell finds correspond to a particular kind of fact.[3] Further, general and existential propositions have particular kinds of facts corresponding to them, general and existence facts.[4] I shall not discuss further these other types of facts and propositions; Russell's discussion can be referred to for a fuller treatment.

A proposition is a complex symbol, whose meaning is derived from that of the component symbols which compose it. It is possible by analysis to reduce propositions to those component symbols, to determine the meaning of these basic symbols, and in this way obtain a clear understanding of the meaning of the original proposition. Since a true proposition corresponds to a fact, a correct analysis of the proposition reveals the components of the fact corresponding to it, for the meanings of the component symbols are the corresponding constituents of the fact the proposition expresses.

We may lay down the following provisional definitions: That the components of a proposition are the symbols we must understand in order to understand the proposition;

That the components of the fact which makes a proposition true or false, as the case may be, are the *meanings* of the symbols which we must understand in order to understand the proposition.[5]

[1] *Monist*, 1919, 37 ff.; *Our Knowledge of the External World*, p. 54.

[2] *Monist*, 1919, 39; but see *ibid.*, p. 201 for a consideration which makes Russell doubtful of the non-existence of molecular facts.

[3] *Ibid.*, pp. 47 ff.

[4] *Ibid.*, p. 200; *Our Knowledge of the External World*, p. 55.

[5] *Monist*, 1918, 518.

The symbols composing propositions 'mean' the corresponding constituents of the facts those propositions express, they 'denote' or 'indicate' or 'stand for' those constituents. The relation of 'meaning' here is that of 'acquaintance' which was discussed in the preceding Chapter.

> All analysis is only possible in regard to what is complex, and it always depends, in the last analysis, upon direct acquaintance with the objects which are the meanings of certain simple symbols.[1]

Atomic propositions are composed of words standing for the terms and relations that compose the corresponding atomic facts, let us then investigate the nature of the 'terms' and 'relations' which compose atomic facts.

The 'terms' in an atomic fact are things that must be known by acquaintance, by direct presentation. What is a term in an atomic proposition is then determined by the limits and extent of our knowledge by acquaintance. Russell calls the words that denote objects with which we are acquainted 'proper names', and so the constituents of atomic propositions that denote terms are 'proper names'. The terms of facts are sometimes treated by Russell in a wider sense as if they were 'things', or 'material objects' in the usual meaning of the word.[2] However, this is a loose use of 'term', for material objects, though they can be treated as terms, are not terms in the strict sense, since they cannot be known by acquaintance. In deciding what it is that can be a term, it might be useful to repeat some of Russell's characterizations of 'acquaintance'. It is: '. . . a direct cognitive relation to that object, i.e. when I am directly aware of the object itself. When I speak of a cognitive relation here, I do not mean the sort of relation which constitutes judgment, but the sort which constitutes presentation.'[3] And: 'We shall say that we have *acquaintance* with anything of which we are directly aware, without the intermediary of any process

[1] *Monist*, 1918, 515.

[2] *Our Knowledge of the External World*, p. 51; *The Analysis of Mind*, pp. 193, 278.

[3] *Proc. Arist. Soc.*, 1911, 108; *Mysticism and Logic*, p. 209.

of inference or any knowledge of truths.'[1] Things that we are acquainted with Russell calls 'particulars'; it is particulars then that are the terms of atomic facts. In 'Philosophy of Logical Atomism', a 'particular' is defined as a term of a relation in an atomic fact (1918, 522), and a 'proper name' is defined as a word denoting a particular (1918, 523). Proper names then denote particulars, those things with which we are acquainted, 'in order to understand a name for a particular, the only thing necessary is to be acquainted with that particular' (1918, 525). How we determine what is *one* particular, rather than a succession of them, or what is a particular rather than a description of an object or a logical construction, is by finding whether it is something with which we can be acquainted. The difficulty is similar to that we found in trying to distinguish between hard and inferred data. As in the case of hard data, Russell came to make particulars more and more limited entities, until they were the momentary sense-data that can be described by such 'names' as 'this' and 'that'. 'That makes it very difficult to get any instance of a name at all in the proper strict logical sense of the word. The only words one does use as names in the logical sense are words like "this" or "that".'[2]

Proper names in a proposition denote particulars in the corresponding fact, general words apparently in some sense denote universals (predicates and relations). I do not find it clear in what sense such universals, or concepts, are denoted; often in Russell's writings it appears as if our knowledge of concepts is of the same kind as that of particulars,[3] but the situation, I believe, is more complicated than this. In the quotation from *Philosophy of Logical Atomism* (my p. 120) concepts and particulars are included as both being 'ultimate simples'. He says of the concept 'red,' 'the word "red" can only be understood through acquaintance with the object . . .'[4] In an earlier paper,

[1] *The Problems of Philosophy*, p. 73; *Monist*, 1918, 523 ff.

[2] *Monist*, 1918, 524; *Philosophy*, p. 256. See also *Introduction to Mathematical Philosophy*, p. 142.

[3] *The Problems of Philosophy*, Chapter X.

[4] *Monist*, 1918, 517.

. . . we can understand universals, and any primitive understanding of a universal (i.e. any understanding not derived from some other understanding) is in a sense a case of acquaintance, and is certainly sufficient ground for calling the universal in question a datum. What is the correct analysis of the understanding of universals is a difficult question . . .[1]

However, in 'Philosophy of Logical Atomism' he says further:

To understand a name you must be acquainted with the particular of which it is a name, and you must know that it is the name of that particular. You do not, that is to say, have any suggestion of the form of a proposition, whereas in understanding a predicate you do. To understand 'red', . . . you understand propositions of the form that '*x* is red'.[2]

These quotations are not necessarily inconsistent, since Russell could maintain that one of the differences between acquaintance with a universal and with a particular is that acquaintance with a universal, unlike that with a particular, carries with it the suggestion of the form of a fact (or of a proposition). The preceding discussion is only a brief summary of some of Russell's views of universals. A more extensive treatment is not necessary at this time.

The correspondence between propositions and facts might lead one to assume that Russell favours a correspondence theory of truth, and such an assumption I believe is correct. A proposition is then true when it corresponds with a fact in the sense already discussed, when every constituent of the proposition corresponds to a constituent of the fact the proposition expresses. A proposition is false when there is no such fact.[3] The discussion which Russell gives of truth is, however, not as simple as this brief statement might suggest. He finds that there are further relations, besides the correspondence of constituents, that hold between propositions and facts.

There are two different relations, as you see, that a proposition may have to a fact: the one the relation that you may call being

[1] 'Definitions and Methodological Principles in Theory of Knowledge', *Monist*, XXIV (1914), 585.

[2] *Monist*, 1919, 34.

[3] *Ibid.*, p. 57; *The Analysis of Mind*, p. 278.

true to the fact, and the other being false to the fact. Both are equally essentially logical relations which may subsist between the two, whereas in the case of a name, there is only one relation that it can have to what it names.[1]

Propositions have a meaning different from that of names, since a proposition can be meaningful to us before we have investigated to see what fact, if any, it expresses. Thus 'Mr. X lives in the tenth house down the street' has meaning to us since we would know where to go and look for Mr. X, yet it might correspond to no fact, and be a false proposition, if there were only nine houses on that street. The meaning of a proposition is derived in some way from the meaning of its component symbols[2] which are either particulars or universals; yet the proposition as a whole has meaning and has the relation of being either true to the fact or false to the fact. At one point Russell suggests a theory prompted by Wittgenstein that true propositions point 'toward' facts, false propositions point 'away' from facts.[3] However, Russell finds this theory unsatisfactory, and the final result seems to be the form of correspondence theory I stated in the preceding paragraph. The reader should not confuse false propositions with true negative propositions, which assert that something is *not* the case; Russell maintains that negative facts exist[4] and true negative propositions correspond to negative facts.[5]

The logical doctrines of this period provide Russell with a framework within which traditional philosophical problems can be analysed. Since every proposition is capable of analysis, propositions concerning various philosophical doctrines can theoretically be analysed to reveal the facts those propositions

[1] *Monist*, 1918, 507–8.

[2] See references given under Note 4, p. 119.

[3] *The Analysis of Mind*, pp. 271 ff.

[4] *Ibid.*, p. 276; *Monist*, 1919, 42.

[5] Earlier, in the *Principles of Mathematics* (see above, p. 22), Russell had maintained that false propositions were subsistent entities, but in the logical atomism period false propositions are only complex symbols. Facts are neither true nor false, but are, of course, entities. These facts include negative facts, which subsist.

correspond to, and the various constituents of those facts. In this way the real meaning of propositions of philosophy can be determined. In particular, Russell is concerned with the problem of the nature of material objects; we can attack this problem by analysing propositions dealing, or purporting to deal, with such objects. Thus 'that cow is brown' at first sight apparently refers to a fact whose constituents are a term, 'that cow', and a one-term relation, 'brownness'. We may assume that 'brownness' is a constituent of the fact, and proceed to investigate the difficulties of considering 'that cow' as a term. If 'that cow' is really a term, then material objects are entities with which we are acquainted, either existent or subsistent. But according to Russell's doctrine of acquaintance, we can be acquainted only with particulars, in this case something approximating 'that brown patch', not with material objects, and the term that seems to be a material object cannot in fact be one. We must find a more adequate analysis of such propositions, one in which we are referring only to particulars and universals.

Such faulty analysis Russell refers to as 'bad philosophical grammar' and he attributes to it many of what he considers the unfortunate results of traditional metaphysics.[1] Previously Russell has shown that if on a superficial analysis symbols representing numbers, objects described, and classes, are considered as denoting terms, those objects then become entities with some kind of metaphysical status. His own analysis has shown, however, that propositions containing such symbols can be reduced to propositions expressing facts not containing any unique corresponding entities, and it is found unnecessary to suppose their existence. The problem for philosophy that Russell states in 'Philosophy of Logical Atomism' is to determine the correct analysis of propositions containing words apparently denoting material objects, and to determine the correct analysis of propositions of physics containing symbols apparently denoting such entities as time, space, and matter. The result of this analysis will show whether such words really

[1] *Monist*, 1919, 365.

denote a special kind of entity, or if those apparent entities are 'constructions' employing only the 'ultimate constituents of the world', particulars and universals, that we already know. Such special kinds of entities, Russell maintains, are 'metaphysical', in the sense that they are never empirically given, as are entities that we know by acquaintance.

> By metaphysical entities I mean those things which are supposed to be part of the ultimate constituents of the world, but not to be the kind of thing that is ever empirically given,—I do not say merely not being itself empirically given, but not being the *kind* of thing that is empirically given.[1]

The aim of this analysis Russell expresses in his 'supreme maxim of scientific philosophizing', which states that all 'inferred' entities should be replaced by logical constructions. 'Metaphysical' entities are 'inferred', since they are not empirically given; if philosophical analysis is successful they will be replaced by constructions, and they will not need to be included among the real constituents of the world. It should be noted, however, that Russell qualifies his maxim, as we have seen him do in his discussion of numbers and descriptions, by saying 'It is possible that there may be all these things that the physicist talks about in actual reality, but it is impossible that we should ever have any reason whatsoever for supposing that there are.'[2]

The elimination of metaphysical entities by the use of logical constructions is made even more desirable as this procedure has the advantage of reducing the 'risk of error' in a system. I shall simply let Russell state his aim here, and assume that the supposition of fewer entities does in fact result in less 'risk of error'. The analysis of propositions which I discussed will then serve this additional purpose.

> What is the smallest number of simple undefined things at the start, and the smallest number of undemonstrated premises, out of which you can define the things that need to be defined and prove the things that need to be proved?[3]

[1] *Monist*, 1919, 368. [2] *Ibid.*, p. 367. [3] *Ibid.*, p. 366.

. . . one thing that our technique does, is to give us a means of constructing a given body of symbolic propositions with the minimum of apparatus, and every diminution in apparatus diminishes the risk of error. Suppose, e.g., that you have constructed your physics with a certain number of entities and a certain number of premises; suppose you discover that by a little ingenuity you can dispense with half of those entities and half of those premises, you clearly have diminished the risk of error, because if you had before 10 entities and 10 premises, then the 5 you have now would be all right, but it is not true conversely that if the 5 you have now are all right, the 10 must have been. Therefore you diminish the risk of error with every diminution of entities and premises. When I spoke about the desk and said I was not going to assume the existence of a persistent substance underlying its appearances, it is an example of the case in point. You have anyhow the successive appearances, and if you can get on without assuming the metaphysical and constant desk, you have a smaller risk of error than you had before. You would not necessarily have a smaller risk of error if you were tied down to *denying* the metaphysical desk. That is the advantage of Occam's Razor, that it diminishes your risk of error.[1]

Certain points, important for the problem Russell wishes to solve, I should like now to discuss more fully. I shall assume the general philosophy of logic that he has developed, in spite of whatever difficulties or inadequacies the reader may feel that it has. The doctrine of 'acquaintance' is fundamental to Russell's discussion, because the terms or 'particulars' that we are searching for in analysis are entities with which we can be acquainted. We need to examine the nature of 'acquaintance', and at the same time the kinds of things with which we can be acquainted. I have previously discussed 'acquaintance' in Chapter II,[2] but it is useful now to discuss the subject further. A quotation from *The Problems of Philosophy* will serve to introduce the discussion.

The particular shade of colour that I am seeing may have many things said about it—I may say that it is brown, that it is rather dark, and so on. But such statements, though they make me know truths *about* the colour, do not make me know the colour itself any better

[1] *Monist*, 1919, 378–9.

[2] See above, pp. 62 ff.

than I did before: so far as concerns knowledge of the colour itself, as opposed to knowledge of truths about it, I know the colour perfectly and completely when I see it. . . . Thus the sense-data which make up the appearance of my table are things with which I have acquaintance, things immediately known to me just as they are.[1]

Acquaintance seems to consist in simply having sensations, or sensible experiences, or 'presentations', the mere occurrence of some sense experience. These, Russell believes, give 'perfect' and 'complete' knowledge, as contrasted with knowledge of truths about sensations, e.g. 'this (the sensible experience) is brown', which is not 'complete' knowledge. It is a rather limited sensible experience with which we are acquainted; the sensible experience which we describe as looking like a table is not a case of acquaintance, as is that of a brown percept. We have acquaintance, according to Russell, only with sense-data or percepts, not with material objects.

The basis upon which we determine precisely what can be known by acquaintance is indicated by Russell's discussion in which he denies that material objects are known by acquaintance:

My knowledge of the table as a physical object, on the contrary, is not direct knowledge. Such as it is, it is obtained through acquaintance with the sense-data that make up the appearance of the table. We have seen that it is possible, without absurdity, to doubt whether there is a table at all, whereas it is not possible to doubt the sense-data.[2]

In the discussion of hard data, I have given some of Russell's reasons for maintaining that material objects are not hard data, and the preceding quotation might very well have served to illustrate that discussion. The grounds for determining what we are acquainted with seem to be the same as those for determining what is hard, rather than soft, data. Both are reached without inference, both are 'certain', in the sense I have discussed in the preceding Part. That which is known by acquaintance, then, is the same as that which in another connection

[1] *The Problems of Philosophy*, pp. 73–4.

[2] *Ibid.*, p. 74.

Russell has called hard data. Granting that this is the case, the discussion of hard data in the preceding Part can be referred to for an indication of the difficulties that would similarly confront the notion of acquaintance.[1]

If the criterion of certainty is not the one for determining the extent of our knowledge by acquaintance, then I fail to discern what the criterion is. It might be maintained that it is a psychological criterion, and Russell's discussion indicates that he himself sometimes thinks of it this way. Thus: acquaintance is 'a direct cognitive relation . . . when I am directly aware of the object'.[2] In this interpretation the doctrine of acquaintance rests upon a psychological theory that we are in fact 'presented' or become acquainted with certain sensations, and then proceed to make inferences from them. Even granting that this is a valid psychological theory, which is surely a debatable point, it does not seem that Russell is concerned primarily with psychology. A psychological theory would be a description of the processes of perception and inference, and does not concern Russell's problem of 'justifying' inferences, of finding the grounds upon which we can logically base our beliefs. This latter problem goes beyond a scientific description and analysis of the conditions and circumstances of the occurrence of perception. Psychology is no more relevant to Russell's discussion of acquaintance than it was to his discussion of hard data, and the further remarks which I gave there equally hold here.

The introduction of psychological elements confuses Russell's discussion of acquaintance and obscures the real importance of

[1] See above, p. 110. It seems plain that in acquaintance Russell refers to actual sensible experiences themselves, not beliefs about them. It is unfortunate that it is not always clear in the discussion of hard data whether Russell meant actual sensible occurrences or beliefs about them. However, there is obviously a close relation between them, since the beliefs in question would be beliefs for which the sensible occurrence would be the only evidence necessary. I believe this distinction could be made clear in Russell's discussion without affecting essentially my assertion that hard data and that which is known by acquaintance are the same.

[2] See above, p. 124.

what he is trying to point out. It has also served to arouse controversy upon points which are aside from the central issue, particularly in debating whether or not Russell's psychology is adequate. It is quite possible that the notion of acquaintance may have been influenced by an inadequate psychology, as the notion of hard data may also have been. It is unfortunate that Russell is not clearer as to precisely the place that psychology did have in his investigations. But, to me, the main issue of the doctrine of acquaintance is not concerned with psychology, but is an attempt to analyse the empirical reference of knowledge. It seeks to reduce propositions to elements present in the individual's own experience, to find analyses of propositions not at first sight concerning such experience that will show that they can in fact be based upon individual experience. In particular, Russell wishes to show that propositions concerning material objects can be reduced to beliefs founded on the individual's sensible experience. But, with this end in view, there is more than one way of analysing our perceptual experience to determine what it is with which we are acquainted, as, assuming Russell's standards of hard data, there is more than one way of determining that data. Whether or not one then grants Russell's notion of acquaintance, it must still be decided whether to accept his standards for determining which objects are known by acquaintance, or to adopt some other manner of determining these objects, such as one I shall suggest in the concluding Chapter.

I have been presenting Russell's views concerning acquaintance with as few of his accompanying metaphysical beliefs as possible. His problem then becomes the analysis of propositions concerning material objects to show that those propositions, or as it turned out, propositions equivalent to them, really denote elements of an individual's sensible experience. It is a problem essentially the same as the one I referred to in the last Part as his 'epistemological problem', and for my purposes it is sufficient to consider them one problem. But, as in the case of hard and soft data, Russell adds further assumptions that alter

the nature of his problem. In the philosophy of logical-atomism this 'metaphysical' problem is definitely I believe the predominating one.

Not only does Russell maintain that what we are acquainted with is our own sensible experiences, but what we know by acquaintance are 'particulars' with the metaphysical status of being part of the 'ultimate furniture of the world'. The discussion in the preceding Part of sensible objects, or 'percepts', which are 'particulars', can be referred to for Russell's beliefs as to the nature of the 'particulars' he is discussing here. Russell's problem now becomes not only to find what sensible experiences our statements about material objects are based upon, but what *particulars*. As in the former case, these metaphysical assumptions may be justified, but they are not results of his epistemological or logical examination. The problem then that Russell finds is to reduce (or replace) the supposed *entity* material object with known *entities*: particulars.

PART III

PHYSICS AND PERCEPTION

The specific cases of the problem of justifying inferred beliefs with which Russell is especially concerned are beliefs in the existence of permanent material objects and beliefs which comprise empirical science. It is the latter class of beliefs which increasingly absorbs Russell's attention in his later writings. An essential part of the problem of justifying scientific beliefs is what he calls the 'interpretation' of science, especially of physics.[1] The desire to find an interpretation for physics is the same aim as he expressed in earlier writings, for example in *Our Knowledge of the External World*, as that of analysing statements of physics concerning space, time and matter to find the ultimate reference of those terms to individual sense experience. I shall discuss this problem of science only briefly as

[1] *The Analysis of Matter*, Introduction; *Human Knowledge*, Part Four, Chapter I.

I think it can be seen that it is a special case of the more general problem discussed in the first Part of this Chapter.

Our least questionable knowledge, Russell believes, is of 'data'; the remainder of our empirical knowledge is inferred, and it is the task of the philosopher to reveal the inferences from data upon which such knowledge is based. If physics, as an empirical science, is to have any claim for our acceptance it too must show how its laws are reached as inferences from data. In Chapter I, I gave one of Russell's characteristic illustrations of the relation between our perceptions and our knowledge of physics. In *Human Knowledge* he discusses this in more abstract terms:

> Every empiricist holds that our knowledge as to matters of fact is derived from perception, but if physics is true there must be so little resemblance between our percepts and their external causes that it is difficult to see how, from percepts, we can acquire a knowledge of external objects. The problem is further complicated by the fact that physics has been inferred from perception. Historically, physicists started from naïve realism, that is to say, from the belief that external objects are exactly as they seem. On the basis of this assumption, they developed a theory which made matter something quite unlike what we perceive. Thus their conclusion contradicted their premise, though no one except a few philosophers noticed this. We therefore have to decide whether, if physics is true, the hypothesis of naïve realism can be so modified that there shall be a valid inference from percepts to physics. In a word: If physics is true, is it possible that it should be known?[1]

Russell does not maintain that we should discard physics because it is inferred, any more than he maintains that we should discard all inferred beliefs (although sometimes in earlier works he implies that the latter is desirable), but it is at least important to exhibit the grounds upon which inferred beliefs rest.

We must, if this enterprise is to be accomplished, provide an 'interpretation' for the abstract system of physics which will serve to relate its laws to the empirical world.

[1] *Human Knowledge*, pp. 197–8; see also *ibid.*, pp. 8, 162.

An interpretation of physics, Russell maintains, is important if physics is to be considered an empirical science; for if it is an empirical science, it must have applications to the empirical world, and above all, its statements must be verified by actual events.

> . . . although physics can be pursued as pure mathematics, it is not as pure mathematics that physics is important. . . . The laws of physics are believed to be at least approximately true, although they are not logically necessary; the evidence for them is empirical. All empirical evidence consists, in the last analysis, of perceptions; thus the world of physics must be, in some sense, continuous with the world of our perceptions, since it is the latter which supplies the evidence for the laws of physics.[1]

An interpretation of physics must serve to 'bridge the gulf between physics and perception'.[2] As he says:

> The evidence for the truth of physics is that perceptions occur as the laws of physics would lead us to expect—e.g. we see an eclipse when the astronomers say there will be an eclipse. But physics itself never says anything about perceptions; it does not say that we shall see an eclipse, but says something about the sun and moon. The passage from what physics asserts to the expected perception is left vague and casual; it has none of the mathematical precision belonging to physics itself. We must therefore find an interpretation of physics which gives a due place to perceptions; if not, we have no right to appeal to the empirical evidence.[3]
>
> . . . in the case of an empirical science the empirical terms must depend upon terms of which the ostensive definition is given in perception. The astronomer's sun, for instance, is very different from what we see, but it must have a definition derived from the ostensive definition of the word 'sun' which we learned in childhood. Thus an empirical interpretation of a set of axioms, when complete, must always involve the use of terms which have an ostensive definition derived from sensible experience.[4]

An 'uninterpreted' system is an abstract deductive system, such as is found in logic, mathematics, or physics, that begins by assuming certain terms, undefined except for properties

[1] *The Analysis of Matter*, p. 6. [2] *Ibid.*, p. 7.
[3] *Ibid.*, p. 7. [4] *Human Knowledge*, p. 242.

enumerated in various primitive propositions, and then from the undefined terms and primitive propositions deduces theorems concerning further relations and properties of the original terms. On the other hand, such a system will be 'interpreted' (according to Russell) when the original undefined terms are replaced by some kind of term having a definite meaning for which all the properties given in the primitive propositions hold. The theorems, then, will state properties and relations which actually hold between these entities.

> It frequently happens that we have a deductive mathematical system, starting from hypotheses concerning undefined objects, and that we have reason to believe that there are objects fulfilling these hypotheses, although, initially, we are unable to point out any such objects with certainty. Usually, in such cases, although many different sets of objects are abstractly available as fulfilling the hypotheses, there is one such set which is much more important than the others. . . . The substitution of such a set for the undefined objects is 'interpretation'.[1]

An illustration of 'interpretation' is Russell's own definition of 'cardinal number'.[2] The uninterpreted system in this case is Peano's axioms for the number system,[3] whose three primitive ideas and five axioms, Peano believed, were sufficient to enable one to derive all the properties of the system of natural numbers. Actually, Russell maintains, Peano's axioms define any progression of the form $x_0, x_1, x_2, \ldots x_n, \ldots$ of which the series of the natural numbers is one instance. To limit Peano's axioms so that they apply to the one particular case of the natural numbers it is necessary to give his primitive ideas the meaning they correspond to in our usual number system. When this is done we will have an 'interpretation' of Peano's axioms. We can also interpret his axioms by substituting for his indeterminate primitive ideas those ideas as they have been defined by Russell in his definition of 'cardinal number', and the result will satisfy Peano's axioms. The system as thus interpreted will

[1] *The Analysis of Matter*, pp. 4–5.
[2] *Ibid.*, pp. 3–4; *Human Knowledge*, p. 236.
[3] See above, p. 30.

state the properties of the natural number system. 'Real' entities have not directly been substituted for undefined terms, but what has been substituted are structures composed of real entities; the ultimate reference of the interpreted system is then to real entities.

An uninterpreted system, in Russell's view, is a system whose primitive terms are variables with various specified properties; when these variables are replaced by constants fulfilling the original specifications, a new system is obtained—an interpretation, or special case, of the general abstract system. An interpretation of the abstract system of physics will give a system whose laws concern empirical events and give a means for manipulating empirical material. Of course, physics is ordinarily assumed to have reference to the real world, but the precise manner in which it does so is not at all obvious. If empirical entities, or structures composed of such entities, can be found to replace such terms as 'matter', 'point', 'instant', then laws concerning these hitherto abstract terms will state relations holding between empirical entities. For example, if we had a system concerned only with 'points', 'point' would be uninterpreted if it were only considered a variable fulfilling certain conditions specified in the primitive propositions of the system. There would be also various interpretations of that system, most of which would be what Russell calls 'unimportant'. The system could be interpreted in terms of the co-ordinates which are the real numbers corresponding to points. 'Point' would then be a set of real numbers. However, to give such a system practical application, the important interpretation is one which defines 'point' in terms of empirical entities.

> It is clear that if geometry is to be applied to the sensible world, we must be able to find definitions of points, lines, planes, etc., in terms of sensible data, or else we must be able to infer from sensible data the existence of unperceived entities having the properties that geometry needs. To find ways, or a way, of doing one or other of these things is the problem of the empirical interpretation of geometry.[1]

[1] *Human Knowledge*, p. 238.

If this is done, propositions concerning points, such as those of geometry, will have reference to actual objects in the world and enable us to apply propositions of geometry to the relations between those objects. It is such an 'important' interpretation which must be found for physics. If this can be found, we will have given the abstract terms of physics a meaning in terms of empirical entities, and can then proceed to determine to what extent the laws of physics are inferences from empirical data.

An interpretation of physics, or of any abstract system, is necessary if that system is to be shown to relate to the empirical world, so that we can show that it is empirically true, or as Russell says, if we are to 'verify' it. In his earlier writings, Russell says, 'verification consists always in the occurrence of an expected sense-datum',[1] and more fully in *Mysticism and Logic*:

> What can we learn by observation and experiment?
>
> Nothing, so far as physics is concerned, except immediate data of sense: certain patches of colour, sounds, tastes, smells, etc., with certain spatio-temporal relations.
>
> The supposed contents of the physical world are *prima facie* very different from these: molecules have no colour, atoms make no noise, electrons have no taste, and corpuscles do not even smell.
>
> If such objects are to be verified, it must be solely through their relation to sense-data: they must have some kind of correlation with sense-data, and must be verifiable through their correlation *alone*.[2]

In *Human Knowledge* we find that 'it is only through sensations that physics can be *verified*'.[3] This can hardly be accepted as an adequate discussion of what is now included under the term 'verification', as Russell's own more extended discussions in *An Inquiry into Meaning and Truth* and *Human Knowledge* show. But these quotations do give one condition which any body of propositions that is to be empirically verified must meet. It might very well be the case that the truth of an isolated

[1] *Our Knowledge of the External World*, p. 81; see *The Analysis of Matter*, p. 8.
[2] *Mysticism and Logic*, p. 145.
[3] *Human Knowledge*, p. 261.

proposition could be established by its relation to other propositions, but that system of propositions as a whole if it is to be empirically verified must ultimately refer to sense experience. This conclusion is hardly surprising if it is remembered that Russell maintains that the least doubtful empirical propositions are those concerning data of perception, while others are inferences based to a greater or less degree on those data. If we are to establish the inferred propositions of science, we must exhibit the data of perception upon which they are ultimately based.

CHAPTER IV

THE CONSTRUCTION OF MATERIAL OBJECTS AND THE ENTITIES OF PHYSICS

In the preceding Chapter it was seen that one aspect of Russell's problem was to justify inferences from data to derived beliefs for purely epistemological reasons, and further, at least in some writings, at the same time to derive a metaphysics. In his writings there is a shift in emphasis which this problem undergoes. In *Our Knowledge of the External World*, as I interpret it, he attempts to use nothing, or at least as little as possible, besides hard data. Justifying beliefs then consists in showing that they are based upon data; if this cannot be done, it is desirable to find an equivalent belief which can be derived from data. But when this has been done, we are left with a strange view of the world. Our world consists mainly of percepts, with unobserved entities eliminated wherever possible.

> Men of science, for the most part, are willing to condemn immediate data as 'merely subjective', while yet maintaining the truth of the physics inferred from those data. But such an attitude, though it may be *capable* of justification, obviously stands in need of it; and the only justification possible must be one which exhibits matter as a logical construction from sense-data—unless, indeed, there were some wholly *a priori* principle by which unknown entities could be inferred from such as are known. It is therefore necessary to find some way of bridging the gulf between the world of physics and the world of sense. . . .[1]

It is for this purpose that Russell analyses such concepts as material object, matter, space and time by means of logical constructions to show that such 'unknown' entities are not

[1] *Our Knowledge of the External World*, p. 101.

required, that classes of percepts can be substituted for such entities. It is the employment of constructions in dispensing with material objects and matter that I shall discuss in Part I of this Chapter.

The result, however, was not entirely satisfactory, for reasons which will become clear. In *The Analysis of Matter*, Russell is less strict about admitting unobserved entities, and although he had employed unobserved percepts in *Our Knowledge of the External World*, he had hesitated to admit events different from percepts. These he accepts in *The Analysis of Matter*. It is still desirable, he feels, not to admit material objects, matter, space and time as single permanent entities, but to construct them as groups of events employing not only percepts but unperceived events as well. The technique of constructions is essentially the same as in the earlier period, and his problem is the same, except for one important qualification. Russell's earlier aim was to derive our inferred beliefs from hard data alone (accepting the principles of logic), but now he maintains one other assumption is necessary besides hard data—the causal theory of perception. I shall discuss these points in Part II.

The construction of points and instants I have kept separate from that of material objects and matter, and I shall discuss them in Part III. In these constructions there is no difference as far as general epistemological and metaphysical considerations are concerned, but the actual technique employed requires a more technical discussion than in the first two Parts.

Finally, in *Human Knowledge*, the attempt to base derived beliefs upon percepts alone, or at least to utilize only a minimum of additional principles, is definitely abandoned. Russell now maintains, as I shall describe in Part IV, that besides data, certain 'postulates' of inference are required to justify the common-sense and scientific beliefs he has been concerned with in his writings. He now must show how derived beliefs are justified using these principles and data, and he must also show what justification we have for accepting these 'postulates'. Constructions are useful to enable us to substitute groups of events

for a single permanent entity representing material object, or point, or instant. It will be seen, however, that it is a question whether they are as useful as Russell formerly maintained they were.

PART I

MATERIAL OBJECTS AS CLASSES OF APPEARANCES[1]

As we have found, Russell believes that in analysing any belief we should start from what we have determined to be hard data and find how the belief in question may be based (if at all) upon that data. In the problem of permanent material objects, let us start by considering the hard data obtained by perception, data that each individual possesses at a given moment. This data, Russell maintains, is a set of percepts from different senses, such as visual data of different colours and shapes spatially arranged, e.g. the shapes and colours of a landscape or the view of a room, and data from such other senses as touch or hearing. An individual's perceptual experience is continually changing, any one view is only momentary. One minute of his experience, for example, would include not only his visual sensations of a room as a whole, but details of the separate pieces of furniture and the people in the room, and in addition percepts of other senses, sound or touch, of the texture of objects in the room and the voices of people conversing. His lifelong perceptual experience is composed of all the views, all the sets of percepts or sense-data, he has ever experienced. The data of each sense form a 'world' peculiar to that sense with spatial relations determined by those data. These worlds, Russell maintains, are immediately given as external, meaning that the data of sense alone determine relations which correspond to

[1] The construction of material objects and matter during this early period can be found in *Our Knowledge of the External World*, Chapters III and IV; *Mysticism and Logic*, Chapters VII and VIII; *The Analysis of Mind*, Chapters V, VII, and XV.

what is normally meant by external spatial relations.[1] The data of sight are given as spatial in sight space, the data of other senses, e.g. touch, are given as spatial in the space, such as touch space, of the sense in question. The one inclusive space of each individual, which makes it possible for him to say that the object he is touching is at the same place as one he is seeing, is a construction from the spaces given by each of the separate senses. It is a construction made by correlating various sensations of one kind with those of another, a correlation learned in early childhood experience.[2]

The momentary set of percepts of each individual, the view of the world which he has at any given moment, Russell calls a 'private world' or a perceived 'perspective'. 'We may therefore define the perspective to which a given particular belongs as "all particulars simultaneous with the given particular", where "simultaneous" is to be understood as a direct simple relation, not the derivative constructed relation of physics.'[3] The definition is carefully phrased because Russell wishes to avoid any reference to a perceiving subject in the definition;[4] as I shall mention shortly he also wishes to define sets of data which are perceived by no one. However, 'simultaneous' as Russell is using it does not seem to avoid reference to a perceiving subject, since apparently it is a relation which depends upon the presence of a subject to whom percepts can be 'simultaneous'.[5]

If, however, perspectives were defined by means of spatial relations, the difficulty would still not be avoided. Russell wishes to include the data from different senses in one perspective, but the relation between the spaces of the different senses is one constructed by the subject. If Russell wishes to define 'perspective' so that its existence will not depend upon a perceiving subject, it will be necessary to clarify the notion

1 *Our Knowledge of the External World*, p. 73.

2 *Ibid.*, p. 113; *The Analysis of Matter*, p. 252.

3 *Mysticism and Logic*, p. 141; *Our Knowledge of the External World*, p. 88; *The Analysis of Mind*, p. 128.

4 *Mysticism and Logic*, p. 140.

5 See *The Analysis of Mind*, p. 128.

of 'simultaneous' so that it will not imply any dependence on the subject. A similar difficulty also occurs in Russell's later discussion of simultaneity between unperceived events.[1] A 'perspective' is all the sets of data, perceived or unperceived, defined by the above definition; 'private world' is a perceived perspective, an individual's actual percepts. All of the percepts perceived during an individual's life Russell calls his 'biography'; those biographies not lived by anyone he calls 'official'.[2]

Confining ourselves to hard data, the only private world that one can be certain exists is his own. But normally it is believed, and Russell adopts the belief, that others have their own perceptual worlds, similar to one another's often, but such that two people at the same time never see exactly the same world. Thus two people sitting next to each other at a ball game will be said to 'see the same thing', but will really at the same time never 'see the same thing', although what they see will be very similar; the one will see the field from a different—even though minutely different—angle. They could not see the same thing, i.e. have the same visual sense-data, unless they were in exactly the same position in space with identical physiological and psychological natures. The admission that others have perceptions similar to ours implies the admission of knowledge by testimony, since we can never ourselves experience another's perceptions. This admission means that Russell is not confining himself to hard data, since facts of testimony, and the belief in 'other minds', is not among our hard data. Russell maintains that though the belief in other minds is not among our hard data, yet it is a natural belief, one never doubted in practical life, a useful, even necessary, hypothesis for action.[3]

One argument, though not one whose conclusion is 'certain', which Russell uses to support this belief is that from the similarity of our behaviour with that of others we can conclude

[1] *Human Knowledge*, pp. 329-30.
[2] *Mysticism and Logic*, p. 141.
[3] *Our Knowledge of the External World*, pp. 93 ff.

that they have percepts similar to ours. In *The Analysis of Matter*, Russell develops this argument in some detail.[1] We observe, he says, correlations between stimulus and reaction (considering both as percepts) in our own bodies, and assume, as an inductive inference, that such correlations will hold in the future. We observe also that the behaviour of other persons is similar to ours in response to various stimuli, and finally we infer from the observation of the behaviour of another person that the usual stimulus to that behaviour occurred, even though we ourselves did not see the stimulus. We are thus led to infer the existence of entities which we do not perceive.

> It will be seen that . . . the argument is the usual causal-inductive type of argument upon which all empirical laws are based . . . the argument for other people's perceptions is the same in form and cogency as the argument for the future truth of laws of correlation among our own percepts. We have exactly as good reason for believing that others perceive what we do not as we have for believing that we shall have a perception of touch if we stretch out our hand to an object which looks as if it were within reach.[2]

The argument for the existence of entities which we do not perceive, though not demonstrative, 'is as good as any of the fundamental inductions of science' and increases the probability of other propositions concerning the existence of other kinds of unperceived existents.[3] Thus either on the grounds of this argument, or by reason of their practical necessity, Russell believes we are justified in assuming the existence of other people's minds. The admission of this one kind of inferred entity, however, is not a ground for admitting further kinds of entities such as material objects, Russell maintains; the former are similar to entities that we experience, while the latter are entities that can never be experienced by anyone.[4] The reader should note that by accepting the belief in other minds, Russell has already abandoned his reliance solely on hard data.

A perspective, whether one's own or that of another person, is the momentary set of sense-data that the individual perceives;

[1] *The Analysis of Matter*, pp. 200 ff.
[2] *Ibid.*, p. 205.
[3] *Ibid.*, p. 206.
[4] *Ibid.*, p. 207.

the individual will have other perspectives with the continual changes in his bodily state or position. As he changes his viewpoint, no matter how slightly, his perspectives will change accordingly. Walking around an object gives him a sequence of perspectives of changing shapes as he sees the object from in front, from the corner, from the side. Admitting the testimony of others, we find that they too can form sequences of perspectives, and can supplement ours, by reporting to us the appearance of objects from positions where we have never been. We can correlate the data in different perspectives and speak of a relation between the perspectives themselves; a perspective which has many of its data similar to corresponding data in another perspective can be said to be 'near' that perspective, while one in which the corresponding data are not as similar, in which a long series of perspectives intervenes between the two perspectives compared, is said to be 'far' from the first. The establishment of a correlation between similar perspectives is the first step in extending our knowledge beyond our immediate sense-data; it gives us in fact a basis upon which we can define 'material object' without going beyond these sense-data, and yet preserving all the functions, Russell maintains, normally associated with the term.

If we consider all the sense-data that normally we say come from the same 'thing', we find that they will be to a greater or lesser extent similar depending upon the nearness of the different perspectives in which they occur. Let us call this group of data simply the 'thing', so that a 'thing' becomes a 'logical construction'.

> By the similarity of neighbouring perspectives, many objects in the one can be correlated with objects in the other, namely, with the similar objects. Given an object in one perspective, form the system of all the objects correlated with it in all the perspectives; that system may be identified with the momentary common-sense 'thing'. Thus an aspect of a 'thing' is a member of the system of aspects which *is* the 'thing' at that moment. . . . All the aspects of a thing are real, whereas the thing is a mere logical construction.[1]

[1] *Our Knowledge of the External World*, p. 89.

Strictly speaking, it is not correct to say simply that a 'thing' is a 'logical construction', but it is more convenient to do so in this exposition. I shall discuss below the more precise sense in which Russell speaks of 'thing' being a logical construction. The construction can be made utilizing only the aspects of the 'thing' actually perceived. The 'thing' would, however, be fragmentary, since it would exist only where and when observers were present, the series of appearances that constituted it would be discontinuous, existing only at positions from which it was actually perceived. It is useful to consider the appearances the object would have from positions where no observers happen at a given time to be present, useful to imagine a continuous change of appearances, permitting us to anticipate the appearance the object would have if we take a position between others we have previously held. We can make the changes continuous by using in the construction elements not perceived, but calculated on the basis of aspects actually perceived. Thus besides all the perspectives that actually are perceived we can, by considering also unperceived perspectives, form a series of perspectives, so that there will be a continuous change in all the data from one perspective to another. This continuity corresponds generally to our actual experience; strictly speaking, Russell believes, the series of perspectives is continuous because it forms a compact series like the series of instants of time or points of space. Empirically, continuity is verified only in a rough form, we cannot be sure how susceptible our observation is to very slight changes. Russell's more inclusive definition of 'thing' employs both perceived and unperceived aspects:

> The 'thing' of common sense may in fact be identified with the whole class of its appearances—where, however, we must include among appearances not only those which are actual sense-data, but also those 'sensibilia', if any, which, on grounds of continuity and resemblance, are to be regarded as belonging to the same system of appearances, although there happen to be no observers to whom they are data.[1]

[1] *Mysticism and Logic*, p. 154.

Each individual's percepts determine their own space, a space private for each observer that is constructed from the spatial relations between the percepts of each of his senses. The space (and time) of material objects is, however, a public space that is not restricted to one particular observer. Russell finds that it can be constructed in a manner similar to the construction of the physical objects in that space. We found it possible on the basis of similarity to correlate neighbouring perspectives, and the percepts composing them, relating perspectives so that some would be 'near', others 'far', from a given perspective. When we are able to relate the position of many of the percepts in one perspective with many in another we can treat the whole perspective as an element and speak of it as being to the 'left', or to the 'right', or 'in front of', the other perspective. Treating perspectives as elements we find a new space is determined, a space in which perspectives are the points—'perspective space'. It is this space, Russell maintains, that is ordinarily referred to in speaking of spatial relations between material objects; it is the space that is examined by physics. It should be noted that perspective space is different from private 'subjective' space, and that each private space is different from every other one; the spatial relations of private and public space cannot be identified. Russell speaks of sense-data[1] being in a 'six-dimensional' space, because three co-ordinates are necessary to determine the position of sense-data in the private space of the observer, and three more co-ordinates are required to specify the position (in perspective space) of that perspective. In *The Analysis of Matter* (252) he says that private and public space have the same structure, that there is a correspondence between the percepts in private space and the material objects, which they constitute, in public space, that the logical properties of the relations in the two spaces are the same.

The following example illustrates the manner in which public (or perspective) space is constructed.[2] Suppose we are

[1] *Mysticism and Logic*, pp. 139, 162.

[2] *Ibid.*, pp. 158 ff.; *Our Knowledge of the External World*, pp. 89 ff.

seeing what we call a 'penny', in one perspective it will appear, as we say, larger than in another, or elliptical in shape instead of round; more precisely, in one perspective we will perceive a round percept which will occupy a larger part of the field of vision than the corresponding percept in another perspective, or we will perceive an elliptical, rather than a round, percept. Suppose we collect all the perspectives which contain a round percept of the penny, i.e. all the views in which the penny looks round, then arrange them in order according to the size of that percept, size again being determined by the proportion of the field of vision occupied by the percept. We can then form a continuous series of perspectives, including both those perceived and those calculated but not perceived, beginning with the one in which the penny 'looks largest' and extending down as small as we wish, since we may remove the penny to a place farther removed and imagine the series indefinitely prolonged. Other series of perspectives can be formed, e.g. a series of views of the edge of the penny composing a continuous series beginning with the one in which the edge looks largest and extending indefinitely toward the smallest. Considering each perspective as an element and the continuous series of them as straight lines or curves, we can form a 'perspective space' in which each perspective is an element or point. The point of intersection of the different lines, the different series of perspectives containing aspects of the penny, is the perspective which 'will be defined as "the place where the penny is" '.[1]

The perspectives that contain a large percept of the penny we can say are 'nearer' the penny than those which contain a smaller percept. There is a limit, however, to the closeness we can get to any object, for we can get no closer than when (by touch) we find the object is touching the eye. Actually, at the place in the space of perspectives where an object is defined to be, there may be no percepts or perspectives, but the place is the point where two series of perspectives if ex-

[1] *Mysticism and Logic*, p. 162; *Our Knowledge of the External World*, p. 91.

tended would intersect. As Russell says[1] the centre of an object in space may be 'hollow', the percepts ranged around that hollow centre. The place 'here' in public space is the perspective that we perceive at the time we say 'here'; if a thing is far from 'here' it means the perspective that defines the place where the thing is in perspective space is far in perspective space from the perspective that is composed of our present percepts. Our percepts can be said to be 'inside our head' since our present perspective is located in part of the place in perspective space that our head occupies. Two places in perspective space are associated with every percept and with every material thing, the place *from* which an aspect of a thing appears is the place of the perspective of which the aspect is a member, the place *at* which an aspect appears is the place of the material thing of which the aspect is a member; or we may say that the place *from* which a material thing appears is the place of the perspective of which some aspect of the thing is a member, and the place *at* which it appears is the place of the thing in perspective space.[2]

In the construction of 'thing', or 'material object', the 'thing' was identified with a class of correlated aspects, the class of appearances of the 'thing'. But on what basis can we collect this class of aspects without employing the notion of permanent objects that the construction is trying to dispense with? Why do we distinguish one series of data and use that to constitute the 'thing', rather than some other series of data?[3] It might seem from the previous discussion as if the continuity between the appearances was the sole basis upon which the class of appearances was selected; so that if we find that similar appearances in different perspectives with the addition of some unperceived aspects can form a continuous series of appearances representing

[1] *The Analysis of Matter*, pp. 211–2.

[2] The reader may find that this distinction clarifies Russell's discussion of the physiologist seeing his own brain. See above, p. 116.

[3] *Mysticism and Logic*, pp. 169 ff.; *Our Knowledge of the External World*, pp. 107 ff.; *The Analysis of Mind*, pp. 125 ff.

'change', or 'growth', or 'decay', the possibility of forming these series will be adequate ground for calling the series a 'thing'. Actually, Russell feels, continuity is neither a necessary nor a sufficient condition for the selection of the class of aspects. It is not a necessary condition since it is in fact not usually observed, and we merely assume that a continuous series could be formed. It is not a sufficient condition since there is often sensible continuity between appearances which we consider as belonging to separate objects. Russell's example is the sea, where there is sensible continuity between the drops, and yet it is desirable to be able to define a 'drop', and to distinguish a current within the water. What is needed in the definition of 'material object' is an expression of the conformity of the series of aspects to laws. The momentary 'thing' (the construction given on my p. 147) can better be defined as those aspects that conform to the laws of perspective[1] using perspective in the sense of my earlier discussion of the construction of things. The continuity between aspects might now be considered a necessary condition for the definition of a thing, if it is believed that the laws of perspective require this continuity. For permanent things, the further condition is required of their conformity to the laws of dynamics. These laws include those correlating the appearances of what is found to be one thing to different senses, correlating the appearances of that thing at different times, correlating the changes in the appearances of the thing as it is influenced by other objects.

Now physics has found it empirically possible to collect sense-data into series, each series being regarded as belonging to one 'thing', and behaving, with regard to the laws of physics, in a way in which series not belonging to one thing would in general not behave. If it is to be unambiguous whether two appearances belong to the same thing or not, there must be only one way of grouping appearances so that the resulting things obey the laws of physics.[2]

To complete the definition of material object (or matter) we

[1] *The Analysis of Mind*, p. 125.
[2] *Mysticism and Logic*, pp. 172–3.

should include a reference to these causal laws in the definition: 'Thus we may lay down the following definition: *Things are those series of aspects which obey the laws of physics.*'[1]

The construction of material object is also a construction that can be used for 'matter', and for this reason in Russell's discussion the terms often are used interchangeably. The difference between the construction of 'material object' and the construction of 'matter' is a difference in the aspects included in the construction, not in the procedure of the construction. Russell is primarily concerned with the general procedure, and feels that the details can be worked out as required; consequently he is usually not concerned with distinguishing between matter and material objects too closely. The definition I have given of 'material object' utilizes *all* appearances of the object, including the views of an object seen from every point of view, at varying distances, and through all kinds of intervening media. In a more precise formulation of 'material object' it might be desirable to limit the aspects which are included in the object to those not seriously affected by the intervening medium (e.g. not to include those views of the object where it is seen through blue spectacles), or to those not at a great distance from the object.[2] Since matter is normally thought to be independent of our sense-organs, and also of the intervening medium, Russell defines 'matter' in *Mysticism and Logic* as the group of aspects taken at as small a distance from the object as possible:

> We commonly assume that the information we get about a thing is more accurate when the thing is nearer. . . . It is obvious that from the point of view of physics the appearances of a thing close to 'count' more than the appearances far off. We may therefore set up the following tentative definition:
>
> The *matter* of a given thing is the limit of its appearances as their distance from the thing diminishes.
>
> It seems probable that there is something in this definition, but it is not quite satisfactory, because empirically there is no such limit

[1] *Our Knowledge of the External World*, p. 110; *Mysticism and Logic*, p. 173.

[2] See *The Analysis of Mind*, p. 106.

to be obtained from sense-data. . . . But probably it suggests the right direction in which to look.[1]

At this point, it might be useful to examine the construction of material objects and matter to see what Russell has accomplished. The preceding theory has been designed to answer the problems that I discussed in Chapter III. I shall discuss several points concerned with the formation of the constructions themselves, and attempt to evaluate their success in answering Russell's problem, and also attempt to determine their validity regardless of the extent to which they serve as an answer to that problem. I presented his problem as first, one predominantly epistemological, to which Russell himself then frequently adds various metaphysical views so making it both an epistemological and metaphysical problem. In the following discussion I shall follow the same distinction as far as it seems to make an evaluation of Russell's analysis more significant. Accordingly the first use of constructions I shall discuss is their employment as an attempt to answer the question of the certainty of the inference from sensible experience to the belief in material objects.

At first sight it appears that the evidence necessary to establish such a belief as 'that is a barn' includes some entity 'barn', and following Russell's analysis it is difficult to see how hard data, that is individual fragmentary sense experience expressed in such beliefs as 'red percept present', can provide the required evidence. It is at this point that the analysis of 'material object' as a logical construction, as an incomplete symbol, becomes significant. The use of logical constructions, Russell maintains, makes it unnecessary to search for such evidence; for statements apparently about material objects, e.g. 'that is a barn', are a kind of circumlocution, simply a manner of speaking. In apparently establishing these statements, we are really establishing the truth of equivalent statements whose nature may be best seen by briefly retracing the route followed in constructing material objects. Beginning from such 'certain'

[1] *Mysticism and Logic*, p. 165.

beliefs as 'red percept present', we find 'certain' beliefs concerning further percepts which it is found can be related to the first percept, allowing us to correlate the different percepts into various series, which, if they are in accord with the laws of physics, Russell calls the 'thing'. If further percepts are found to be members of one of these series, they will belong to the 'thing'. We can now assert not only 'red percept present', but '[this] red percept member of class of correlated percepts'. But this, Russell maintains, is all that is needed to establish statements concerning material objects, since statements such as 'that is a barn' are equivalent to 'red percept member of class of correlated percepts'. Russell has not replaced the *entity* 'material object' with a group of percepts, but he maintains that statements apparently referring to material objects are equivalent to statements referring to classes of appearances, namely, those used in the construction of material objects. It is this latter kind of statement that he has been trying to show can be inferred from percepts, but since these statements are supposed to be equivalent to the former kind, when they are established statements apparently referring to material objects will also be established.

The employment of logical constructions in Russell's analysis of beliefs concerning what are commonly called material objects has not, nor does he maintain that it has, made these beliefs hard data. The individual sensible occurrences which form the material of the construction are hard data, but it must be shown further that they belong to a class of percepts by establishing the required correlations. The assertion that this correlation does exist, or that a given percept is a member of the class of correlated percepts, is an inferred belief, as Russell conceives of 'inference', since it is a conclusion resulting from the evidence of a considerable number of beliefs. This evidence is not complete because of the inclusion in the class of percepts of unperceived aspects whose actual existence has not been verified. Though the belief has not been made hard data, what has been accomplished is that the inference has been made

explicit, with the grounds upon which it is based. This represents a definite advance over the belief that material objects exist as entities, an inference to an unperceived entity whose grounds, Russell maintains, cannot be discovered.

Besides the desire to answer the question of the 'certainty' of our beliefs in material objects, Russell also wished to find the real denotation of symbols apparently standing for material objects. Since material objects as constructions are like other constructions in being 'incomplete symbols', symbols such as 'barn' do not denote a class of correlated percepts, but statements containing such symbols are equivalent to other statements which contain symbols denoting only percepts and universals. In this way, Russell maintains, statements concerning material objects have empirical reference. Statements apparently referring to an entity, 'material object', do not require us to find such an entity to verify our statement, but are verified when we verify the equivalent statement by finding the correlated percepts that are members of the class constituting the construction.

The question that the employment of constructions so far has been an attempt to answer can be considered primarily as an epistemological one rather than as both an epistemological and metaphysical question, because they have been used in determining the grounds for asserting statements equivalent to 'that is a material object'. I have maintained that this can be done without beliefs as to the metaphysical nature of the various materials used. What we begin with on Russell's view are certain beliefs concerning our immediate sensible experience, that can be interpreted as patches of colour, noises, touch sensations, what might be approximately expressed as 'brown patch occurring', without any determination of the metaphysical status of 'brown patch'. This evidence gives us ground for inferring, Russell believes, statements such as 'this brown patch is a member of a certain class of percepts', which again need carry no metaphysical implications. These statements are equivalent to statements that such and such

is a material object, and hence serve the same purpose. Not only are metaphysical assumptions irrelevant for this enterprise, but for the reasons I discussed in the preceding chapter it is not only irrelevant to introduce psychological considerations into the formulation of the problem, but similarly irrelevant to introduce them in the solution. Thus Russell's occasional statements that we do *in fact* perform the constructions he advances as an analysis would seem to be beside the point.[1]

In evaluating the construction of material objects and matter, it is important to examine the extent to which they follow Russell's own standard of basing inferences upon hard data alone. There are two types of data employed that are questionable. The first is data obtained through testimony. It is not difficult, I believe, to see the necessity for their inclusion, but the fact that they are necessary reflects upon the validity of the conception of hard data which taken strictly excludes them. The need for data from testimony can be shown by considering what constructions would be without their use (and, for the present, without the use of unperceived aspects). Constructions would then use as data only that actually perceived by one person, and the result would be roughly that 'that is a material object' is equivalent to 'that percept is a member of a class of my perceived percepts' where the class has been established by correlations between one person's percepts. If we lived in our own private world this might be an adequate formulation; it might be possible to verify all our statements about material objects on a solipsistic basis.[2] But this in fact is not what is done; for our belief that something is a material object implies that others when properly situated will obtain perceptions similar to ours. In short, we mean that others 'can see the same

[1] See, for example, the discussion above on pp. 143–44, where the construction of the one inclusive space for each individual is a correlation learned in early childhood. In *Our Knowledge of the External World*, p. 104, the construction of public space is treated as a psychological fact.

[2] *The Analysis of Matter*, p. 213.

thing'. Verifying statements about material objects requires that the reports of the data of others can be correlated with our own data. Russell follows the customary meaning of 'material object' in including such data in his construction. Even if the question of verification did not arise, it is necessary to include the use of testimony if we are to have common communication. The reports of others will mention data which correlate very closely with our own, and frequently are presumed to refer to the 'same' object as that to which our data belong. But unless the reported appearances are included in the same class of appearances as constitute the object we are referring to, they will not belong to the same object. They will otherwise belong to the class of appearances out of which the other observer has constructed the object, an object constituted only by his data. If we do not accept data from testimony there will be no material object common to different observers, but each will construct his own object, different from that of all the others. Thus Russell's construction is so made that common reference is possible about what is observed, but the data admitted to make this possible are not hard data.

The second kind of questionable data included in the construction is what Russell calls 'unperceived aspects' or 'sensibilia'. Their employment offers no great difficulty as long as we discuss the epistemological aspects of the problem; they become more questionable when we consider their metaphysical status, but that is not my present concern. Again, we may consider what the construction of material objects would be like if these 'sensibilia' or 'ideal elements' were not included. In that case statements concerning material objects would be equivalent to statements concerning data actually perceived by various observers. Thus if I wish to establish the statement that there is furniture in the next room, I can go into that room accompanied by other observers and establish the equivalent statement that the perceptions of the other observers and myself could be correlated in the proper manner. If, however, there were no observers in that room, then there would be no

class of percepts, and it would be impossible to make any statements about the furniture there. This is not the normal conception of material objects. The assertion that something is a material object implies that certain roughly predictable future percepts will be obtained when specified conditions are fulfilled; a material object is such that *if* we fulfil certain conditions we will receive such and such sensations. One step in verifying the existence of such objects is to determine what would be the perception under certain conditions not yet realized, then realize those conditions and see if in fact the perceptions are forthcoming.

The discussion of the 'continuity' of perception which Russell gives is an expression of the need for unperceived aspects. As we move around any object, although we in fact receive sensations from only a few positions, we believe that if this object is a material object we should be able, in principle at least, to obtain a continuous series of perceptions. Russell has also made this point in the discussion of the verifiability of physics.[1] For an enunciation of the laws of physics, and to show how they are at least capable of being verified, we should be able to calculate how things will appear where there is in fact no observer, or how they will look at a time when no one is observing them, or how things will look which are not observed at all. Without the use of unperceived aspects there will be none of the continuity and constancy of reference in beliefs about material objects that both common usage and Russell desire. The furniture in the next room would be constituted by the class of percepts of the observers present, if no one were present for a time, upon re-entering the room statements made about the furniture would not be about the same furniture as before because a new class of percepts would be referred to. Similarly, future percepts from an object would not be possible, since they would result in a new class of percepts, and consequently comprise a different object from the one we assumed they were. To avoid such difficulties, Russell includes 'hypothetical', or

[1] *Our Knowledge of the External World*, pp. 110 ff.

'ideal' percepts in his construction, the appearance that an object *would* have if such and such were the case.

Statements about material objects are then equivalent to statements concerning a class of correlated percepts; they are inferences that are based on beliefs of which some are 'certain', which are known to the person in question, some founded on testimony which are not certain, but are essential, and finally, some concern unperceived, but calculable (or 'ideal') percepts. The 'calculation' in the latter is a calculation which offers, I feel, little difficulty, since it is one performed frequently in everyday life. Thus I am standing now at the door of a room surveying the furniture in it, and I can visualize reasonably well what it will look like from different positions within the room, or if its present position is changed. 'Ideal' percepts are not certain in the sense that one's actual perceptions are; they are inferences on the basis of present perceptions, memory, and testimony. Russell's word 'calculate' in referring to them is an indication of this element of inference, since their presumed appearance is an estimate, an approximation on the basis of known data.

Hypothetical percepts are calculated by inferences, and hence are not hard data. In cases where they are based on the individual's own percepts they are inferences on the basis of hard data. It is difficult to determine the extent to which they violate Russell's standard of employing hard data. If Russell is interpreted to mean that the class of percepts in the construction is to be *entirely* hard data, and the class of beliefs upon which the construction is based *all* certain beliefs, then, of course, the inclusion of unperceived aspects though warranted for practical reasons violates Russell's standards. But if we interpret Russell to mean that not only the construction itself is an inference based on hard data, but that data employed in it may be inferences, provided they are inferences based on hard data, the use of unperceived aspects might be admissible. In either case, regardless of the validity of the inclusion of unperceived aspects, data from testimony are not hard data since

they are inferences to unobservable events. The use of constructions has not, whatever its virtues may be, succeeded in showing that assertions, which Russell maintains are equivalent to assertions about material objects, can be established by 'certain', or 'non-inferred', beliefs alone.

As long as we are considering only the problem of the validity of the inference to material objects, without regard to their metaphysical nature, it is sufficient to consider unperceived aspects as hypothetical, as 'ideal'. The inference depends upon beliefs from actually obtained percepts, and upon beliefs that predicted percepts will be obtained. E.g. if our belief that there is furniture in the next room is true, then upon entering that room we will receive certain calculable percepts. If these percepts are not received, then our original belief in the existence of a material object was false. To verify statements concerning material objects we require then statements concerning predicted, but not realized, percepts. The reader will often find that Russell's statements go beyond this characterization of unperceived aspects and attribute to them metaphysical status of some kind, but this indicates his preoccupation with the metaphysical aspects of his problem I shall discuss shortly. Some of his statements, however, appear to imply that their hypothetical existence is sufficient; thus he maintains that the construction employing hypothetical appearances, and considering them only as hypothetical, will satisfy formally all the demands required of constructions.[1]

The use of beliefs which are not certain in the construction of material objects shows that Russell has not reduced the inference to material objects, or the inference to the equivalent statement he substitutes for that belief, to certain beliefs. Granting that the construction of material objects does result in statements equivalent to those apparently referring to an entity, material object, the inference to such constructions is possibly better grounded than that to the entity material

[1] *The Analysis of Matter*, p. 213; *Our Knowledge of the External World*, p. 112; *Mysticism and Logic*, p. 158.

object. For if we grant Russell's initial analysis of knowledge, 'material object' refers to something unlike anything conceivably experienced in sensation, and consequently inferable only on 'risky' grounds, while his construction, though not based wholly on 'certain' beliefs, at least requires (besides logical principles) the assumption of nothing totally unlike anything in our experience.

I have so far been concerned with the adequacy of Russell's analysis in finding statements equivalent to statements purporting to refer to material objects and attempting to base them on hard data. It might, however, very well be the case that the two types of statement are equivalent, even though they do not solve Russell's problem. His analysis would then be valuable as an analysis of the essential factors involved in our knowledge of material objects, regardless of the extent to which one accepted Russell's initial beliefs concerning empirical knowledge. But if the two types of statements are found not to be equivalent, this would mean that Russell's analysis of the notion of material object or matter is not adequate, and would also be a further reason for questioning that analysis as a solution for his own problem. Unfortunately, it is not easy to judge whether the two are equivalent; the difficulty is indicated by the problem faced by other philosophers who have attempted to analyse 'material object'. I shall not make the attempt to evaluate the adequacy of logical constructions as an analysis of 'material object' myself, but to suggest that the difficulties philosophers have faced who have advanced theories dispensing with objective entities such as material objects in favour of percepts would lead one to the trial of all possibilities of analysis which will avoid the difficulties encountered by the commonsense notion of material object, and yet not require its complete elimination.

One consideration which does, however, I feel reflect on the adequacy of Russell's analysis is the assumption of the laws of physics in formulating the construction. The series of appearances which Russell employs to replace material objects are

those which conform to the laws of physics. But on what basis are the laws of physics established? It seems in fact that the laws of physics have been established assuming the existence of material objects, or matter, or recently some more sophisticated concept of something objective to the observer. If Russell's procedure is to avoid circularity, the laws of physics must first be established solely on the basis of percepts, and then used to construct material objects and matter. It may be quite possible to carry out this procedure, but it would be desirable for Russell to show that it is possible. I have not found this point discussed in Russell's writings.

In deciding what aspects are to be included in one 'thing' we can easily enough keep in the back of our minds the familiar common-sense object we are constructing and choose those aspects which we ordinarily say belong to that object. Russell himself frequently speaks as if this was what he was doing, but it is necessary to show that this is not merely a convenient way of speaking; for he should show that we can forget what we know about the object, and proceeding from appearances alone construct it. If statements which are ordinarily assumed to refer to material objects are really equivalent to statements referring to classes of appearances, then we should be able to dispense with the former in all references to material objects. If this is not the case, then the two statements are not equivalent and Russell's analysis is not adequate.

Regardless of the extent to which Russell's analysis is adequate, and his solution of the problem he has set for himself adequate, he has made the valuable effort of suggesting an analysis of material objects and of the grounds for our belief in them. The relation between our perceptual experience and our beliefs that elements of that experience concern what we call material objects is not at all obvious; Russell has realized the importance of a more precise determination of that relation, and proceeded to investigate it. Although the success of his enterprise I feel is subject to the qualifications I have mentioned, he has made clear several important distinctions that are essential

to the analysis, and provided an analysis that, however adequate, must be taken into consideration in examining the problem.

The problem I have been discussing up to this point I have largely confined to one of epistemology. I shall discuss now the more extensive problem which constructions were designed to solve when metaphysical theories as to the nature of percepts are included in Russell's analysis of knowledge. He wishes to determine what entities (if any) exist that are inferable from objects of sense, but not given in sensation, and specifically, whether it is possible to infer the existence of material objects as entities. He finds that constructions make it possible to dispense with material objects as entities, so that it is not necessary either to merely assume their existence, or base it on some 'risky' inference. Statements concerning material objects, which at first sight look as if they denote an entity, material object, are found to be equivalent to statements which are found to denote only percepts and universals. Our 'certain' beliefs, which assert the existence of various definite percepts, do not have to serve as the ground for an inference to an unobserved entity material object; but instead, to show that a percept is 'of a material object', it is necessary only to show that it belongs to a class of correlated percepts.

The metaphysical view of the world which we obtain, granting the adequacy of the construction of material objects, is one which contains percepts and universals. It does not contain material objects; for although Russell says he cannot prove that they do not exist, yet on the basis of his construction there seems little reason to suppose their existence, or to see what their function would be. If, for the present, we do not consider 'ideal' percepts, it is a world which reminds one of Berkeley (aside from the presence of universals); for, although Russell emphasizes the fact that his percepts are physical, in the brain, while Berkeley's are mental, in the mind, I cannot feel that this is as significant a difference as

Russell, in *Our Knowledge of the External World*, feels it to be.[1]

I carefully excepted 'ideal' elements in comparing Russell's view to Berkeley. The question of the metaphysical status of ideal elements is one which quite understandably gives Russell considerable difficulty. 'Ideal' elements are necessary in the construction, and for the epistemological analysis, it is satisfactory to let them remain 'ideal.' But if percepts now are real entities, what is the status of these unperceived yet necessary 'ideal' entities? In contrast to the references I cited previously[2] in discussing the 'hypothetical' status of unperceived aspects, when Russell is primarily concerned with metaphysical issues he says that ideal elements have the same 'metaphysical and physical status as sense-data without necessarily being data to any mind.'[3] In *Mysticism and Logic* he calls these objects 'sensibilia,' and says that sense-data are perceived sensibilia. Thus we have a world composed of particulars, some of which are perceived and are sense-data, the remainder are like sense-data, except that they do not happen to be perceived, and possibly never will be.

Attributing reality to unperceived elements avoids the implausibility of a fragmentary world composed only of perceived aspects, since without that reality the world would exist only when and where there is an observer present. The difficulty now is if unperceived aspects are supposed to resemble sense-data, and sense-data are dependent upon the body of the observer, it would seem that the particular sense-datum perceived cannot exist until the observer is there to perceive it. However, '. . . we can reasonably suppose that *some* aspect of the universe existed from that point of view, though no one

[1] Percepts, though not mental, are still within the body, and it is a class of these percepts that serves the same purpose as a material object, which is commonly thought to be outside, independent of the subject's body. Both Berkeley and Russell deny the existence of the latter.

[2] See above, p. 161, note 1.

[3] *Mysticism and Logic*, p. 148; see *Our Knowledge of the External World*, p. 88.

was perceiving it.'[1] The main difficulty, to my mind, with this view is that the notion of an unperceived sense-datum is a contradiction in terms; although it might enable us to avoid a Berkeleyan world, invoking an unperceived percept to do this seems hardly a reasonable solution. The point is not worth carrying too far, since I am not certain that Russell did maintain that the only particulars are sense-data, although this is the impression he usually gives at this time.[2] One might interpret an unperceived sense-datum as an entity, such that when brought into contact with a sense-organ becomes a sense-datum. I can find, however, no evidence that this was Russell's belief at this time. Even if this were Russell's position, it would have its own difficulties, since the entity that would become a sense-datum has properties very similar to the material object with which Russell wishes to dispense. It is not conceivably a perceivable entity, it is 'metaphysical' in Russell's sense of never being given empirically, and consequently its use is contrary to Russell's expressed aim during this period.

By the use of logical constructions it is now possible for us, Russell maintains, to adopt a metaphysics that can dispense with entities for which there is no way of finding logical justification. Unfortunately his metaphysical view, if we confine ourselves to actual percepts, omitting 'ideal' percepts for the present, is very similar to Berkeley's, and whether or not Berkeley can be refuted, his views have been found implausible by the majority of philosophers, not to speak of ordinary people, since his time. And implausibility is a good objection, though not of course a demonstrative argument, against any metaphysics. If we add to Russell's view the belief that ideal aspects are real entities, the situation is not improved; for I find it difficult to make such entities acceptable.

[1] *Our Knowledge of the External World*, p. 88; *Mysticism and Logic*, p. 150. See also Russell's 'Reply to Criticisms', in *The Philosophy of Bertrand Russell*, pp. 708–9.

[2] *Mysticism and Logic*, p. 157: 'A complete application of the method which substitutes constructions for inferences would exhibit matter wholly in terms of sense-data . . .'

The success of constructions in their more metaphysical use is somewhat dubious. First, the employment of constructions is supposed to enable us to avoid inferences to 'unknown' entities and to base our knowledge largely, if not entirely, on 'known' entities. My previous discussion indicates the extent to which I believe his construction is not based solely on 'known' entities (or hard data), and hence he has not succeeded in eliminating inferences to unknown entities. However, Russell might reply that the entities he has used are better known than the supposed entity, material object. Secondly, aside from the degree to which Russell's analysis answers his own problem, is the metaphysics which results an adequate one? If we exclude unperceived aspects from his construction, statements ostensibly referring to material objects would be equivalent to statements referring to classes of perceived sense-data. But the inadequacy of Berkeley's metaphysic shows that the two statements are not equivalent, that we mean much more by 'material object' than a class of perceived sense-data. If we include unperceived aspects in the construction, I find the difficulty with the metaphysical status of unperceived aspects such as to make it impossible to determine what the meaning of statements referring to classes of perceived and unperceived aspects is. As a result of these difficulties, which Russell himself fully realized, he revised his construction of material object.

PART II

MATERIAL OBJECTS AS CLASSES OF EVENTS

The constructions that Russell formulates in *The Analysis of Matter* and adopts in later works not only include, as the earlier ones had, perceived and unperceived percepts, but further unperceived events as well. In this Part I shall discuss his views as they are found principally in *The Analysis of Matter* and *Philosophy*,

with reference, especially as his metaphysics is concerned, to *The Analysis of Mind*.[1] 'Events' are existing particulars, of which percepts are now one class. The reason for their employment primarily arises from Russell's desire for a more adequate metaphysical view than the earlier constructions provided. The additional events which he now employs provide, he hopes, a world more nearly in accord with common-sense views, one that exists apart from and independently of any observer.

> I find myself constitutionally incapable of believing that the sun would not exist on a day when he was everywhere hidden by clouds, or that the meat in a pie springs into existence at the moment when the pie is opened. I know the logical answer to such objections, and *qua logician* I think the answer a good one. The logical argument, however, does not even tend to show that there are *not* non-mental events; it only tends to show that we have no right to feel sure of their existence. For my part, I find myself in fact believing in them in spite of all that can be said to persuade me that I ought to feel doubtful.[2]

In the earlier version of constructions, 'appearance' is apparently used as meaning 'sense-datum', either one in fact perceived, or one unperceived, but 'calculated'. The transition from the earlier to the later version is illustrated by the following passage from *The Analysis of Mind*, where 'appearance' is used in an almost metaphorical sense, referring not only to sensations, but to entities correlated with sensations, such that when brought into contact with a sense-organ they are followed by the appropriate sensation:

> Confining ourselves for the moment to the psychology of perceptions, we observe that perceptions are certain of the appearances of physical objects. From the point of view that we have been hither-

[1] See *The Analysis of Matter*, Chapters XX through XXVII, especially Chapters XX, XXIII, and XXVII; *The Analysis of Mind*, Chapters V, VII, and XV; *Philosophy*, Chapters XI through XV, and XXVI.

[2] *Philosophy*, pp. 290–1. This statement contrasts strangely with the aim expressed in many of Russell's works that it is precisely such beliefs that philosophy should investigate. Cf. *Philosophy*, Chapter I.

to adopting, we might define them as the appearances of objects at places from which sense-organs and the suitable parts of the nervous system form part of the intervening medium.[1]

In *The Analysis of Matter*, 'sensation' or 'appearance' returns to its former, more usual meaning, and Russell uses 'event' to refer to particulars that are not perceived.

One justification for admitting unperceived events is that since, Russell holds, we have found it necessary to admit data from testimony and unperceived percepts, '. . . there seems no reason to draw the line at the precise point where it was drawn by Berkeley. On these grounds I feel no shame in admitting the existence of non-mental events such as the laws of physics lead us to infer.'[2] The fact that we have already admitted some unperceived percepts is not, however, according to Russell's own earlier standard, sufficient reason to admit further kinds of unperceived events. We have seen that Russell maintains that the argument for the percepts of others, which are the same kind of entity as our own percepts, is stronger than the argument for unperceiv*able* events, and we cannot indiscriminately classify the different kinds together.[3]

A more adequate justification for the belief in unperceived events is found elsewhere in *The Analysis of Matter*. The general principle which he argues for there upon which to base unperceived events is the causal theory of perception. This theory in Russell's version holds, briefly, that aside from subjective factors influencing our perceptions, external events 'cause' our perceptions, and that the existence of these causes can be inferred from the perceptions. On the basis of the causal theory of perception Russell feels the existence of events causally correlated with perceived aspects to be probable. He accepts the causal theory of perception as a theory that cannot be disproved and as a useful hypothesis in explaining the world. The causal

[1] *The Analysis of Mind*, p. 104; see also *ibid.*, pp. 101, 134.

[2] *Philosophy*, p. 291. Also: 'If we have once admitted unperceived events, there is no very obvious reason for picking and choosing among the events which physics leads us to infer.' *The Analysis of Matter*, p. 325.

[3] *The Analysis of Matter*, p. 207; see above, p. 146.

theory, he feels, is in conformity with the common-sense view of perception that there are occurrences which 'cause' our perceptions, and which furnish the similarity and constancy in that perceptual experience. For example:

> Science holds that, when we 'see the sun', there is a process, starting from the sun, traversing the space between the sun and the eye, changing its character when it reaches the eye, changing its character again in the optic nerve and the brain, and finally producing the event which we call 'seeing the sun'. Our knowledge of the sun thus becomes inferential; our direct knowledge is of an event which is, in some sense, 'in us'. . . . There is the assertion that [perception] has external causes as to which something can be inferred from it. . . . [The theory] depends upon postulates which have little more than a pragmatic justification. It has, however, all the merits of a good scientific theory—i.e. its verifiable consequences are never found to be false.[1]

It is primarily on these pragmatic grounds that Russell accepts the theory.[2]

His acceptance at this time of the causal theory does not mean that he finds the need for constructions is any the less. The causal theory does not imply that there is *one* cause for the different sensations of an 'object', it does not require the existence of a permanent material object. The permanent material object is still a construction to Russell, only now from a set of events, including events causally continuous with the percepts of that 'object'. We need not, except for convenience, talk about the 'cause' of our perceptions, but interpret material objects as classes of percepts and events correlated and continuous with them.

On the basis of the causal theory of perception we can infer the existence of unperceived events correlated with our sensations; we have, however, no ground for inferring anything as to the intrinsic nature of these events, since we only experience percepts and have no way of knowing the character of other kinds of events. What we can infer is that differences between

[1] *The Analysis of Matter*, p. 197.

[2] See *ibid.*, pp. 199, 217.

percepts correspond to differences between the events causing them, but whether these causes do or do not resemble percepts we cannot know.

We assume that differences in percepts imply differences in stimuli—i.e. if a person hears two sounds at once, or sees two colours at once, two physically different stimuli have reached his ear or his eye. This principle, together with spatio-temporal continuity, suffices to give a great deal of knowledge as to the *structure* of stimuli. Their intrinsic characters, it is true, must remain unknown; but we may assume that the stimuli causing us to hear notes of different pitches form a series in respect of some character which corresponds causally with pitch, and we may make similar assumptions in regard to colour or any other character of sensations which is capable of serial arrangement.[1]

Our inference, then, is that percepts and the events causing them are similar in structure, an inference made on the basis of the assumption that a complex cause and effect have the same structure. Similarity of structure is a notion which I have discussed in giving Russell's definition of cardinal number.[2]

This similarity of structure is shown in the relation between private and public space,[3] where the percepts in private space 'correspond' to the objects in public space of which they are part. In private space, the spatial relations between aspects will correspond to relations between constructed objects in public space, e.g. if one percept is between two other percepts, the corresponding objects will have the same relation in the same order. Thus, the properties of unperceived events we can infer are their logical and mathematical properties, since 'when two relations have the same structure . . . all their logical properties are identical'.[4] Strictly speaking, it is probable that the relation between percepts and unperceived events is rather that of 'semi-similarity', than similarity.[5] A semi-

[1] *The Analysis of Matter*, pp. 226–7.
[2] See *Ibid.*, Chapter XXIV.
[3] *Ibid.*, p. 252; see above, p. 149.
[4] *Ibid.*, p. 251.
[5] *Ibid.*, p. 254.

similar relation is a many-one, rather than a one-one, correlation between two systems. Thus although there is a different cause for every different percept, different causes may not necessarily give different percepts:

> We find often that indistinguishable percepts are followed by different effects—e.g. one glass of water causes typhoid and another does not. In such cases we assume imperceptible differences—which the microscope may render perceptible. But where there is no discoverable difference in the effects, we can still not be sure there is not a difference in the stimuli which may become relevant at some later stage.[1]

The group of percepts, ours and other persons', can now be enlarged by events which are correlated with percepts by causal laws. A physical object, or piece of matter, becomes a set of events arranged around what is probably a hollow centre, containing no events; included among these events are our percepts, those of other observers, and events at places in space where there are no observers. Russell believes that there is still no need to assume a permanent substance that in some way gives rise to these events, but the group of events collected according to causal laws is sufficient for all the uses that the permanent substance served.

On the basis of the present form of constructions Russell advances a metaphysics fundamentally different from that presented in the preceding Part. He discusses it much more extensively than he had the earlier form, and I shall attempt within the next few pages to present his views.[2] He calls his new metaphysics 'neutral monism', since he finds it similar in many respects to the neutral monism of Mach, James, and the American new realists.[3] According to this view, there is no dualism between two separate substances, mind and matter, but there is rather only one substance, and mind and matter are

[1] *The Analysis of Matter*, p. 255.

[2] *Ibid.*, Chapter XXXVII; *The Analysis of Mind*, Chapter XV.

[3] For an earlier, critical view of Russell's on 'neutral monism', and the philosophers mentioned above, see 'On the Nature of Acquaintance', *Monist*, XXIV (1914), section on 'Neutral Monism', 161 ff.

different arrangements of this one substance. Some dualistic views maintain that physical objects are permanent physical substances possessing certain properties which give rise to various sensations upon being perceived, such sensations as colour, sound, taste, that exist only in the mind, and are mental. In the early formulation of the construction of matter in *Our Knowledge of the External World* and *Mysticism and Logic*, although Russell still maintains the distinction between mental and physical, he is one step on the road toward monism by not assuming that matter is a permanent substance. He found that sensible effects can be considered physical, even though private to each individual, and that physical objects can be constructed from these sensible effects and other unperceived appearances correlated with them. In the later constructions, physical objects become groups of events, or particulars, some of which (percepts) exist within the brain, some of which exist outside the brain. What would be left for mind, then, or the perceiving subject, is the awareness of these percepts. In *The Analysis of Mind*, Russell abandons the distinction between a percept and the awareness of that percept, and finds that there need be no such entity as 'mind' or 'consciousness', but that perceptions, images, and the laws relating them can account for our 'mental' phenomena. Sensations, perceptions, and images are all events, some of them also parts of physical objects; it is their distinctive correlation by laws peculiar to psychology that characterizes 'mind'. Mind, then, is also a construction; and a construction from events as are material objects.[1]

In attempting to find the distinguishing feature of mind, Russell investigates, and dismisses, the notion of 'consciousness' as a separate entity, or as the essential element in 'mind'. E.g. he no longer analyses sensation into a sensible object and the awareness, or consciousness, of that object, but into sensations and accompanying sensations or images; the 'consciousness' of a sensible object is its leading to further perceptions or

[1] I shall discuss here only the general features of Russell's construction of mind; a more detailed account will be found in *The Analysis of Mind.*

images. Instead of attempting to determine the nature of mind by analysing 'consciousness', he finds it is simpler to utilize the two ways of classifying particulars that he has found. Particulars are members of two groups: (1) the group which is the physical object of which the particular is a member; (2) the perspective of which that particular is a member.[1] Two places for every particular are associated with this classification: (1) the place in perspective space of the object of which the particular is a member; (2) the place in perspective space of the perspective that contains the particular. A perspective, or a set of appearances related by 'simultaneity', is essential for a mind, since a mind at least consists of sensations from one point of view, what might be called 'subjectivity'. But Russell finds that this is not a sufficiently distinguishing characteristic of mind, for even a photographic plate[2] has this kind of subjectivity, it is a collection of appearances seen from one place. The occurrence of images might be thought to be a distinctive characteristic of mind, as images are often considered to be something purely 'mental'. But Russell finds that this also is not sufficient; for in their intrinsic nature images do not differ from sensations, they are as much a part of the physical world as sensations:

> Images are just as truly part of the actual world as sensations are. All that we really mean by calling an image 'unreal' is that it does not have the concomitants which it would have if it were a sensation . . . the distinction between images and sensations can only be made by taking account of their causation. Sensations come through sense-organs, while images do not. . . . An image is occassioned, through association, by a sensation or another image, in other words that it has a mnemic cause—which does not prevent it from also having a physical cause. . . . Sensations, on the other hand, will only have physical causes.[3]

Images in themselves may not be of much assistance in defining mind, but the fact that although they are events like other

[1] *Our Knowledge of the External World*, p. 92; *Mysticism and Logic*, pp. 139, 162; *The Analysis of Mind*, p. 129. See also above, pp. 149 ff.

[2] *The Analysis of Mind*, p. 296.

[3] *Ibid.*, pp. 148–51.

'physical' events, they are subject to different laws, may give us a clue. What distinguishes an image from a sensation is its connection with other images and sensations by different laws of causation than those that hold between sensations and the outside events with which they are correlated. An image has the same intrinsic quality as a sensation, but unlike a sensation, it is not a member of a group defining some physical object. It is then 'unreal', and the laws of causation holding between physical objects do not hold for it. Images have their own laws of production, the kind that psychology has treated extensively in its discussion of the association of images and sensations, in the relation of images to our bodily states and to each other. It is sufficient, then, Russell believes, to dispense with the notion of a 'subject', or 'consciousness', or 'mind', as a separate entity, and to consider 'mind' to be simply the group of sensations and images between which the laws peculiar to psychology hold.

Physics and psychology then both deal with the same material, events, of which one kind are sensations or images. Some events, however, will be usually only treated by physics, say events at some distance from any observer, and some treated only by psychology, sensations and images.[1] But we have no reason to believe that the intrinsic nature of these events is not the same. Both the physicist and psychologist are interested in their own percepts, the psychologist in the particular percepts and their inter-relations within the perspective in which they occur; the physicist is interested in percepts, together with whatever events can be inferred from them, as members of a group constituting a physical object. Thus when Russell says, 'I should say that what the physiologist sees when he looks at a brain is part of his own brain, not part of the brain he is examining',[2] he means that what we see are percepts, located in our brain; our knowledge of the 'external' physical world is our perception of these percepts together with whatever inferences are justified on their basis.

[1] *The Analysis of Mind*, pp. 25–6.
[2] *The Analysis of Matter*, p. 383.

We can now understand the distinction between physics and psychology. The nerves and brain are matter: our visual sensations when we look at them may be, and I think are, members of the system constituting irregular appearances of this matter, but are not the whole of the system. Psychology is concerned, *inter alia*, with our sensations when we see a piece of matter, as opposed to the matter which we see. Assuming, as we must, that our sensations have physical causes, their causal laws are nevertheless radically different from the laws of physics, since the consideration of a single sensation requires the breaking up of the group of which it is a member. When a sensation is used to verify physics, it is used merely as a sign of a certain material phenomenon, i.e. of a group of particulars of which it is a member. But when it is studied by psychology, it is taken away from that group and put into quite a different context, where it causes images or voluntary movements. It is primarily this different grouping that is characteristic of psychology as opposed to all the physical sciences, including physiology; a secondary difference is that images, which belong to psychology, are not easily to be included among the aspects which constitute a physical thing or piece of matter.[1]

The newer form of constructions is designed to solve the same problems for which the previous one had been developed, and the evaluation of the results now obtained can presuppose the discussion of the accomplishments of the previous constructions. I brought out several limitations of the earlier form in fulfilling Russell's epistemological aims, especially in the use of data from testimony and hypothetical percepts. The newer constructions substitute, for statements presumably referring to material objects, statements referring to classes of events, which include perceived percepts, percepts from testimony, as well as unperceived events correlated with percepts on the basis of causal laws. Not only will these constructions be subject to the limitations which I have maintained affect the earlier ones, but the additional one introduced by including events that are not themselves percepts, but only causally correlated with them.

The epistemological purpose of constructions is to determine

[1] *The Analysis of Mind*, pp. 301–2.

in what manner statements referring to unperceived entities may be justified, especially those concerning material objects, and if not justified, how such reference can be avoided. The inclusion of unperceived events in constructions makes Russell's result essentially circular, since in showing that statements referring to classes of events are equivalent to statements about material objects, many of the events are based on inferences of the same kind as those which are often used to infer the existence of material objects. The inference that Russell originally questioned was that which, beginning from statements referring to percepts, inferred statements referring to material objects that cannot be perceived. In the earlier constructions Russell maintained it was possible to avoid such an inference by substituting for it statements referring only to percepts of one kind or another. But now the statements which he substitutes require beliefs referring to unperceivable events. How are statements referring to such events justified? Russell's ground is apparently the causal theory of perception, which in turn he adopts for pragmatic reasons. But on the basis of this theory, held for the same reasons, one might very well maintain the existence of material objects. The difference between the common-sense material object and Russell's events is not an epistemological distinction based on the different grounds for belief in them, but a metaphysical difference between one thing and a class of particulars. One might very well maintain, as does Russell, that the pragmatic grounds in favour of the causal theory of perception are very strong, but it is precisely theories based on such grounds as this that Russell believes philosophy should analyse in order to determine whether a sounder logical basis can be supplied for them. In short, by accepting the causal theory of perception for the reasons which he gives, Russell seems to have lost sight of the goal which he set for himself, and for philosophy in general.

I have maintained that the new form of constructions does not provide Russell with an adequate solution for the primarily epistemological aspects of his problem. Do these constructions,

however, enable him to reach an adequate metaphysics? Beginning from what Russell considers to be 'known' entities, he hopes to find grounds either for inferring the existence of additional entities, or to find ways by which such inferences can be avoided. This latter alternative the use of constructions, he believed, made possible. His aim, as I interpret it, is to provide a more logical basis for a metaphysics than merely to adopt one because it gives a plausible interpretation of one's experience. He is not in sympathy with establishing a metaphysics on the basis of 'pure reason', but to attempt to establish one on the basis of inferences from whatever entities we can be sure that we know. The best 'known' entities, as it will be remembered, are percepts. In the earlier constructions Russell attempted to admit only 'known' data, but as we have seen departed from this standard to accept entities known only by testimony and unperceived percepts. The metaphysics which resulted was not one which Russell himself felt to be wholly satisfactory, for the reasons which I discussed in the preceding Part. But with the new constructions, the world is composed of events, a considerably more plausible world than the earlier form. Unfortunately, although his metaphysics is more believable, constructions employing unperceived events do not really support the inferences to that metaphysics. I have discussed why I believe that Russell's construction is circular, that the inclusion of unperceived events is the very inference which he is trying to justify. The same considerations would affect his constructions in their more metaphysical employment, the inclusion of unperceived events is the kind of metaphysical inference which Russell is concerned to examine and provide a more adequate foundation for.

We can, however, evaluate neutral monism as a metaphysics without regard for the extent to which constructions have given it a sound logical basis. To do this requires an extended discussion which I do not wish to undertake here. Neutral monism is not at all so clearly inadequate as Russell's earlier metaphysics was, and its examination is a much more difficult matter. I

all merely refer the reader to a classic favourable treatment the general position, and to various criticisms of Russell's rm of neutral monism.[1] Russell's contribution to the position, lmitting that the use of constructions has not been of assistance providing a logical justification for the position, is that nstructions enable events to be grouped in different ways to rve the functions for which the conventional 'mind' and natter' have been employed. Russell has formulated definite ocedures for determining which groups of events can be lled 'matter', which can be called 'mind'.

The final result of Russell's analysis from one point of view unfortunate. If we begin with statements referring only to ard data, to our own percepts, we cannot establish beliefs quivalent to those in material objects. If we add to this the etaphysical assumption that percepts are particular existent ntities, then the metaphysical conclusions which are inferred om this are unsatisfactory. But on the other hand, if we do ot restrict ourselves to hard data, but admit other kinds of ercepts and events, the metaphysics which results is more tisfactory, but we have failed to justify the inferences involved. he situation might be avoided in several ways. Granting that ussell's original problem in some form is valid, as I believe at it is, the gratuitous metaphysical assumption that percepts re particular existent entities can be modified, or better, even iscarded. Further, the conception of what it is that constitutes ne basic beliefs from which we begin can be modified in veral directions, one of which I shall suggest in the concluding hapter.

[1] A classic statement of 'neutral monism' in another version is William mes, *Essays in Radical Empiricism*, 1912. Criticisms of Russell's 'neutral onism', can be found in A. O. Lovejoy, *The Revolt Against Dualism*, 1930, hapters VI and VII, and in the essays by Boodin, Laird, and Stace in *The Philosophy of Bertrand Russell.*

PART III

THE CONSTRUCTION OF THE ENTITIES OF PHYSICS

Although the analysis of matter and material objects is an important problem in its own right, and one of Russell's major concerns, it is only one of many similar problems. I have interpreted Russell as maintaining the importance of the examination of inferences from basic beliefs to unobserved entities. Such entities include not only matter and material objects, but many others, especially unobserved entities of use in science. For the philosophy of science it is important not only to analyse the nature of the inference to 'matter', but also those to 'points' and 'instants', since the latter notions are intimately related to that of matter. Russell finds that the procedure he employed in the analysis of matter and material objects, that is, by means of logical constructions replacing statements presumably referring to such objects by statements referring only to percepts and unobserved events, can be applied to these further notions, and others, as well. It will be sufficient here to discuss only Russell's construction of 'points' (and the more complex notion of 'point-instant') for the essential procedure of the construction of other scientific entities is the same.

The general method which Russell uses to construct points and instants from percepts and events is not difficult to show; the details of his solution, however, vary from time to time. In *Our Knowledge of the External World* he constructs points by a procedure whose originality he credits to Whitehead.[1] This method was later improved by Whitehead himself, and called by him the method of 'extensive abstraction'. In the second edition of *Our Knowledge of the External World*[2] Russell adopts the improvements which Whitehead had made. Later, however, in *The Analysis of Matter*[3] Russell finds objections to White-

[1] *Our Knowledge of the External World*, p. 114.

[2] *Ibid.*, second edition, p. 123 n.

[3] *The Analysis of Matter*, Chapter XXVIII.

head's method, and abandons it to return and develop a second method which he had previously outlined in the first edition of *Our Knowledge of the External World*.[1]

The method of defining points that Russell uses in *Our Knowledge of the External World* may be illustrated by the following example: In demonstrations of geometrical propositions, a point is indicated by a dot on the blackboard; it is the smallest area that can be drawn and is supposed to suggest position without extension. A larger area drawn on the blackboard with a more definite shape would not be considered a point, but could 'approach' one if it is gradually made smaller and smaller until it becomes a dot. Let us imagine this process continued beyond the stage where the chalk can make no smaller mark, and beyond the smallest visible surface our eye can perceive. We will then have a series of decreasing areas, included in one another, extending beyond the smallest perceptible area. Considering this as an infinite series of decreasing areas, each member being included in the next larger one, and each having some extension, no member can be found which we can say has the smallest area. We choose, then, any member of the series, no matter how small, and still find a smaller one included in it. This series approaches, but never quite reaches, a point, a position without extension, since every member has some extension and there is no smallest member. We might define a point as the limit of this series of decreasing areas; the limit would itself not be a member of the series since it has no area, but it would be approached, although never reached, by that series. In this way, it might seem that we could define a point in perceptual terms, that is, as the limit of a series of included areas.

[1] *Our Knowledge of the External World*, first edition, Chapter IV, pp. 114 ff; his later method is given in *The Analysis of Matter*, Chapter XXVIII, and *Human Knowledge*, Part Four, Chapters V, VII, and X. The principle of extensive abstraction is in A. N. Whitehead, *An Enquiry Concerning the Principles of Natural Knowledge*, Cambridge, 1919, Part III, and *The Concept of Nature*, Cambridge, 1920, Chapters III and IV. A good clear discussion can be found in C. D. Broad, *Scientific Thought*, New York and London, 1923, Chapter I.

The difficulty with this possible definition of 'point' is that we have merely replaced the unknown entity 'point' with an equally unknown 'limit'. It is a difficulty analogous to that encountered by Russell in accepting a definition of irrational numbers as limits of series. There is no way of perceiving this limit of the series of areas, since it is of a different nature from those areas; but even worse, we cannot prove that this limit exists. This definition is not then adequate for Russell's purpose. We can, however, avoid these objections by defining points, not as the limit of the series of areas, but as that series itself. In this case a point is defined as the set of included areas, those areas which commonly would be said to include the point. So far, I have defined a point in two dimensions, but it must also be done for three dimensions, for a point in space. The illustration can be modified for this purpose by supposing a series of included volumes, such as a nest of Chinese boxes, each within the other. Assuming this to be an infinite series of included volumes, similar to the infinite series of included areas, then a point is defined as the set of included volumes, those volumes which would be said to include the point.

The actual definition of a point which Russell gives is considerably more complicated than the brief outline I have just given; I shall give a statement of the definition indicating only the important factors to be accounted for. The relation of 'enclosure'[1] is a transitive, asymmetrical relation, and such that a single spatial object always encloses itself. Any set of spatial objects such that there is at least one spatial object enclosed by all of them has a lower limit, or minimum, that is, an object enclosed by all of them and enclosing all objects which are enclosed by all of them. Instances of such a relation of enclosure exist. The relation of enclosure with these properties is a 'point-producer'. A set of objects will form an 'enclosure-series' if, of any two of the objects, one is enclosed by the other. To make this series converge to a point, we must add the further condition that our enclosure series is a series such

[1] *Our Knowledge of the External World*, p. 115.

that, given any other enclosure series of which there are members enclosed in any arbitrarily chosen member of our first series, then there are members of our first series enclosed in any arbitrarily chosen member of our second series. Further, any object which encloses itself also encloses an object other than itself, thus providing infinite divisibility. A series meeting these conditions will be a 'punctual enclosure-series', and a 'point' is defined by Russell as the set of all objects which enclose members of a given punctual enclosure-series.[1]

Although I am primarily concerned with presenting Russell's construction of 'points', the discussion of his definition of 'instants' in *Our Knowledge of the External World* is relevant, since one of the two methods he used in that definition he later developed in *The Analysis of Matter* and used for a definition of points, instead of the one I have just presented. In the first method, instants can be defined by a procedure similar to that used in defining points. Russell maintains that we immediately experience two time relations between events, that of simultaneity, or that of earlier and later.[2] We can define an enclosure-series between events, in which one event is temporally enclosed by another, when the former event is simultaneous with the latter, but not before or after it. Proceeding in the same manner as in the definition of points, Russell defines 'instant' as the set of all events which enclose members of a given punctual enclosure series.[3]

The second method, however, is that which is further developed in *The Analysis of Matter*. Finite events, like chalk marks, occupy space, and have duration. Of any two events, the first may be earlier than the second, simultaneous with it, persist after the second has ended, or may have all three relations. A finite event cannot indicate a position without duration in time, any more than a finite area can indicate position without extension in space. But we can approach an ever decreasing

[1] For a discussion of the adequacy of this definition of point for the geometrical use of the term, see Broad, Whitehead, op. cit.

[2] *Our Knowledge of the External World*, p. 116. [3] *Ibid.*, p. 121.

duration by starting with two overlapping events, of which one ends before the other ends, and take a third event which is simultaneous with both of the first two. This third event exists only during the time when the first two overlap, and we have now a shorter duration, a more precise position in time, than if we had referred only to one of the first two. If we continue on in this manner, taking more and more events, the new ones all being simultaneous with those previously collected, the new events will have less and less extended duration, and we will be closer to what we mean by an 'instant'. Russell then defines an 'instant' as a set of events with the following properties:[1]

(1) any two of the events of the set overlap, so that there is some time, however short, when they are all simultaneous;

(2) any other event simultaneous with those of the set is to be included in that set;

(3) no event outside the set is simultaneous with all of them, but all the events of the set are simultaneous with each other.

The enclosure-series method of defining points is essentially Whitehead's method of extensive abstraction, and Russell's criticisms of that method can also be taken as criticisms of Whitehead. It is not necessary for my purpose to discuss Whitehead's employment of the method of extensive abstraction, nor whether it is adequate for the purposes for which he intended it. Russell, however, finds the method unacceptable for the definition of points and instants primarily because of the following reasons:[2]

(1) The method, Russell says, proposes to start with only actual constituents of the world; but the enclosure series of events are series which have no lower limit, or no minimum size of events. The absence of such a lower limit is something whose existence cannot be verified; although we have no direct or indirect evidence for its absence, we have also no

[1] *Our Knowledge of the External World*, p. 118; *Human Knowledge*, pp. 271 ff.
[2] *The Analysis of Matter*, pp. 291 ff.

conclusive evidence against the absence of such a lower limit. Because of our inability to prove or disprove the existence of such minimum events, it would be preferable not to require the assumption of their existence.

(2) Whitehead also assumes, Russell says, the absence of an upper limit to the series of events, or no maximum to the size or duration of events. The objections to this are largely practical; since we are interested in analysing the structure of the physical world, if we take events extending over considerable distance and time we obscure whatever differences exist within the events.

Normally, we try to differentiate events into parts, also events, which are of such a duration that we can discern no further distinguishable parts. Such events are, of course, quite short. Again, it would be preferable it we did not have to use events of great extension in our definition.

A method which avoids either the assumption of the absence of a maximum or a minimum size to events Russell finds by developing the method he used in defining instants as classes of simultaneous events. An additional reason for the development of this method is that in *The Analysis of Matter* he is confronted by a difficulty that he did not have to face in *Our Knowledge of the External World.* It is now no longer 'points' and 'instants' that must be defined, assuming a separate spatial and temporal order, but 'point-instants' for the space-time of relativity. The clearest presentation of his method is to begin with it as it is applied to the one-dimensional time order.[1] Russell defines the relation of 'compresence' between events, which is essentially the same relation as the previous one of 'overlapping' between events in time. In one biography, if a group of events are all compresent with each other, they will be at some place in space-time, and this place will be a point if there is no event

[1] In *Human Knowledge*, pp. 279 ff., Russell briefly develops this method for 'points' as they occur in classical physics. In a later chapter he introduces the relation of 'compresence' in discussing 'point-instants' in space-time, pp. 329 ff.

outside the group compresent with all of them. We may then define a 'point-instant' in one biography as a group of events with the following two properties:

(1) Any two members of the group are compresent;
(2) No event outside the group is compresent with every member of the group.[1]

In extending this procedure to more than one dimension, considerable difficulty is encountered in defining compresence, or overlapping, for high dimensions. Briefly, what Russell does is to introduce the relation of 'co-punctuality' and define 'point-instant' for these dimensions similar to the manner in which it was done for one dimension:

As the fundamental relation in the construction of points, we take a five-term relation of 'co-punctuality', which holds between five events when there is a region common to all of them. A group of five or more events is called 'co-punctual' when every quintet chosen out of the group has the relation of co-punctuality.

A 'point' is a co-punctual group which cannot be enlarged without ceasing to be co-punctual.[2]

From this definition, Russell proceeds to develop definitions of such concepts as lines and surfaces.[3] His procedure is similar enough to that employed in the definition of point-instants to make any discussion of these further definitions unnecessary, and whatever comments I have to make on the definition of point-instants will apply equally to these further definitions.

In evaluating the construction of points, one factor should first be clarified. Russell has maintained that his construction of the entities of physics is necessary as an 'interpretation' of physics and geometry. If this is the case, it might seem that the construction has not shown that the assumption of the existence of these entities can be avoided, but has only 'interpreted' them. This matter can be explained by distinguishing between points and point-instants as they occur in a deductive,

[1] *The Analysis of Matter*, p. 295.
[2] *Ibid.*, p. 299.
[3] *Ibid.*, Chapter XXIX.

uninterpreted system, and as they occur in an interpreted system which is designed to have empirical reference. The construction has not affected the entities as they occur in the uninterpreted system, they still possess the kind of existence, not, of course, physical, that entities in an abstract deductive system of logic or science possess.[1]

Points and point-instants occurring, however, in an interpreted system would have properties different from those characterizing them in an abstract system. Propositions of geometry concerning points, interpreted as having empirical reference, might imply that points are some kind of entity having actual existence, or derived in some way from such entities. It is point as an entity in this sense that Russell believes constructions can avoid. Instead of assuming that there are real points in a real space, statements apparently referring to such entities are equivalent, if Russell's analysis is accepted, to those referring only to classes of events.

It is unnecessary to give an extensive discussion of the adequacy of the construction of points and point-instants.The problems which they were devised to solve are essentially the same as that for which matter and material objects were constructed, the procedure is basically similar, and they are both subject to the same limitations. The construction of the entities of physics are supposed to provide an interpretation of physics in empirical terms, or 'to bridge the gulf between physics and perception', to substitute classes of percepts for unobservable entities as the construction of material objects was supposed to do. But, as in the case of the construction of material objects, by employing unperceived percepts and unperceivable events these other constructions are clearly not an interpretation in terms of empirical, or observed, entities. They possibly have the same value as the construction of matter in replacing one permanent entity with a group of events, but the inferences that support the belief in the latter are no better grounded by constructions

[1] I believe this distinction is clear from Russell's writings. He also made this point in personal conversation.

than that in the former. They cannot, then, as I see it, serve to give the inferences to physical entities any surer foundation.

Even though I have maintained that the construction of points and other scientific entities does not solve Russell's problem they still might be of value as an analysis of those entities. They are clearly ingenious analyses, but I find it somewhat difficult to discover what has been gained by them. They do not reduce the entities to observable elements, nor do they help 'validate the inferences to physics'. It is also questionable to what extent they correspond to the common meanings of the terms involved, or to the way in which the scientist actually uses those terms.[1]

PART IV

POSTULATES OF SCIENTIFIC INFERENCE

In *Human Knowledge*, there is a modification of Russell's problem, and correspondingly, in the use of constructions. Russell attempted, in *Our Knowledge of the External World*, to base our knowledge as much as possible upon hard data, and constructions were found to be valuable since they enabled him to dispense with various beliefs, such as those in the existence of a permanent material object, which he thought could not be justified as inferences from data. The result, however, left him with a view of the world which he came to feel should be rejected because of its implausibility, and we have seen that in *The Analysis of Matter* he admits unperceivable events to restore a more normal picture of the world. I maintained that employing unperceivable events in the construction of matter means that the construction can now no longer be employed as a substitute for inferences to the external world, since those inferences are used in the construction itself. But now in *Human Knowledge*, Russell explicitly maintains that he does not want to dispense with inferences to the external world of events, that

[1] See Ernest Nagel's essay in *The Philosophy of Bertrand Russell*, pp. 343 ff., for a criticism of the value of the construction of points.

the problem is to determine what, besides data, are required to justify them. He finds that besides data we require several principles of non-deductive inference, which he calls 'postulates', to justify that knowledge.

There are three positions which Russell begins his argument in *Human Knowledge* by distinguishing: (1) what he calls the 'solipsist' position, which asserts that nothing is really known beyond one's own immediate data; or (2) we can make inferences beyond our own immediate data to unperceivable events on the basis of certain principles of inference; or (3) some intermediate position, such as Berkeley's or Russell's own in *Our Knowledge of the External World*, in which some inferences, say those to other persons' minds, are admitted, but inferences to unperceived events not admitted. Russell first argues that this last position is untenable.

Hard data, in which no element of inference can be detected, are fragmentary and of definitely limited extent. Even filling out a sensation by 'interpretation', or inferring closely similar sensations, is going beyond hard data. This limited knowledge is all that can be admitted, Russell maintains, if we reject all principles of inference and accept the solipsist position. In *Our Knowledge of the External World* he admitted much more than data, especially the percepts of other persons and unperceived percepts, but he did not admit unperceived events. He rejected the latter because he believed they were even further removed from our sensible experience than unperceived percepts, and represented a more doubtful inference than the latter. It was an inference even less grounded than that to other persons' minds. This position Russell now criticizes on the ground that the inference to other persons' minds is based on some non-deductive principle of inference, and is no different in kind from the inference to unperceived events. Both are based on some non-deductive principle, and if we admit one, why not admit the other?

But this view, since it admits the experiences of others than myself, and since these experiences are only known to me by

inference, considers that it is possible to argue validly from the existence of certain occurrences to the existence of others; and if this is admitted, it will be found that there is no reason why the inferred events should be experienced.[1]

Unlike his earlier view, Russell sees now no reason for discriminating between entities on the ground of differences between kinds of non-deductive inferences. No belief, or event, based on such a principle can be data, and we must either reject all of them, or find some principle justifying those inferences; if we succeed in justifying the inference to one type of unperceived event, we can justify the inference to any type.

We are then left with either of the first two possibilities, we are confined to our own data, which leaves us with a world of fragmentary percepts, or we must admit certain principles which justify inferences to other persons' sensible experience and to unperceived events. The former alternative Russell rejects, as he has previously rejected it, chiefly because solipsism is 'psychologically impossible to believe'.[2] And further, 'when we begin to reflect, we find ourselves with an unshakable conviction that some of our sensations have causes external to our own body'.[3] The usual considerations which show solipsism, no matter how sound a view logically, to be unbelievable are sufficient reason, Russell feels, for dismissing it as a possible alternative.

The reason for rejecting solipsism is first that it contradicts basic common-sense notions, and further, because it denies the picture of the world which Russell maintains science gives us.[4] What science maintains, according to Russell, is essentially the world as composed of the series and classes of events as discussed

[1] *Human Knowledge*, p. 181; see also *ibid.*, p. 177.

[2] *Ibid.*, p. 180.

[3] *Ibid.*, p. 226.

[4] 'There are some who would deny that physics need say anything about what cannot be observed; at times I have been one of them. But I have become persuaded that such an interpretation of physics is at best an intellectual game, and that an honest acceptance of physics demands recognition of unobserved occurrences.' 'Reply to Criticisms', in *The Philosophy of Bertrand Russell*, p. 701.

in *The Analysis of Matter* where there are relatively independent chains of events ranging outward from centres. Our perception of an 'object' is the last event in one such chain. A world organized in such a way preserves, according to Russell, the truth of physics. The reader might very well question whether such a view is in fact what science maintains, but this question I shall take up later. We can obtain this view of the world, and thus remain in harmony with scientific theory, by adopting the second alternative, that there are principles justifying our inferences from percepts to unperceived events. In this case, starting from percepts, we can infer the unperceived events and correlate them into chains. In *The Analysis of Matter*, Russell justified these inferences by invoking the causal theory of perception, but in *Human Knowledge* he gives a more thorough analysis and adopts instead several general principles which he believes are sufficient.

To understand the principles which Russell introduces, it will be convenient to restate the view of the world that science gives us in a different form. First, the traditional notion of permanent material objects seems to be demanded by the relative permanence of objects and people in the world. The notion assumes that there are permanent objects, or indestructible pieces of matter, which persist for some—often extended—period of time. When the results of recent physics are taken into consideration, this notion can be more validly maintained if we do not assume that any event is permanent in the strict sense of the word, but instead speak of 'quasi-permanence' among events.[1] Thus very frequently, Russell says, given any event, we find at a neighbouring time a very similar event in a neighbouring place. To use a crude illustration, a house is not considered to be composed of one or more events which persist until it is destroyed, but composed of series of events, so that those which compose it at one moment are not the same, but are very similar, to those which compose it at a shortly earlier or later moment. 'Quasi-permanence' is a

[1] *Human Knowledge*, pp. 458, 487.

feature of both science and refined ordinary experience. It is preserved in Russell's construction of 'object' where the groups of closely similar events would all be included among members of the class of events constituting the 'object'.

Secondly, we find that the world consists of chains of events, to a large extent independent of each other. These chains range outward from a centre; our percepts are the last events in such chains.[1] Referring to the previous illustration, a house will include a group of events around a centre, with chains of events leading off from this centre. Certain of these chains will compose what are called light waves, and if an observer is present one of these chains will be terminated in his percept of the house. Certain considerations aid in making the existence of these chains seem reasonable. In the first place, percepts, such as visual ones, can be modified by the intervening medium, fog, coloured glasses, or innumerable other conditions, and this can be explained because a chain of events of a certain type is affected by a different kind of event and the succeeding members of the chain transformed. Besides, there are measurable lapses of time between the beginning and the end of a chain, which correspond to the length of the chain. Chains of events are 'independent' in the sense that one chain can be traced from the observer to one centre, but they are not completely independent since one chain may be affected, as in the preceding illustration, by other chains. Russell says chains of events form what he calls 'causal lines'.[2] 'A "causal line", as I wish to define the term, is a temporal series of events so related that, given some of them, something can be inferred about the others whatever may be happening elsewhere.'[3] It is not necessary to discuss the word 'cause', its usage here is sufficiently indicated by the quotation.

Thirdly, chains of events are also continuous, without gaps or jumps which would produce 'action at a distance'. Events will

[1] *Human Knowledge*, pp. 204, 228.
[2] *Ibid.*, pp. 453 ff., 481–2, 489.
[3] *Ibid.*, p. 459.

not influence other events which are completely separate in time and space, unless there can be some intervening chain. Physical objects will continue to exist, even when they are not perceived; there will be events continuous with the perceived group of events, so that an object later perceived will be the 'same' as one formerly perceived since the events later perceived will be continuous with the former. In constructing physical objects, Russell employed continuous series of percepts, or events, as part of the object.

Fourthly, we believe there are also common, objective, 'objects', or groups of events which are the common origin of the percepts of various observers. Russell gives the illustration of several observers and several motion picture cameras watching what we say is the same performance of a play. The observers describe closely similar percepts, and the cameras will be found to have recorded something closely similar to what the observers reported. It is possible to assume that these various records and groups of percepts arose spontaneously and independently of each other and external events, but the more usual belief is that some central system of events is the origin of these similar groups of percepts.[1] Not only do we have causal lines of events from a centre to our own percepts, but many such lines from the same central group of events to different observers, whose percepts will be similar, correlated according to the principles of perspective and changes in the intervening medium. It is this notion which provides a public world. In all of these considerations, it should be noted that 'similar' has the same sense as in *The Analysis of Matter*, where it meant similar in terms of structure. Whether our percepts resemble earlier events in the chain causing them in any other sense is a question which Russell says we cannot answer; we can, however, be reasonably convinced that such events are alike in structure.[2]

These four characteristics of events have resulted in a world similar to that Russell found in *The Analysis of Matter*. The

[1] *Human Knowledge*, pp. 460 ff.
[2] *Ibid.*, pp. 230–1, 296, 467 ff.

construction of things and matter is the same in both works, but in *Human Knowledge* is only briefly referred to,[1] is assumed without being developed. The significance of discussing events from these four characteristics is that Russell introduces a 'postulate' to justify each of the characteristics,[2] besides a fifth postulate to justify the use of analogy, especially as it occurs in the inference to the percepts of others. By the use of these postulates, we are justified in our inferences from data to the series and classes of events which are the picture science gives us of the world.

We now are faced with the question of the justification of these postulates themselves. They cannot, Russell maintains, be logically deduced from experience, nor can we, in fact, find any way by which they can be definitely proved.[3] But their great usefulness is that, if true, they make the inferences from perception to the laws of science valid.

> But we most certainly do need *some* universal proposition or propositions, whether the five canons suggested . . . or something different. And whatever these principles of inference may be, they certainly cannot be logically deduced from facts of experience. Either, therefore, we know something independently of experience, or science is moonshine. It is nonsense to pretend that science can be valid practically but not theoretically, for it is only valid practically if what it predicts happens, and if our canons (or some substitute) are not valid, there is no reason to believe in scientific predictions. . . .
>
> If they are not in fact true, the things that we expect will not happen. They may be approximate, and usual rather than invariable; but with these limitations they must represent what actually occurs.[4]

Yet Russell in an earlier passage in this work states: 'All that strictly follows is that our inferences as to physical objects are consistent with experience, but there may be other hypotheses

[1] *Human Knowledge*, pp. 230, 458, 488.
[2] *Ibid.*, pp. 487 ff.
[3] *Ibid.*, p. 507; *An Inquiry into Meaning and Truth*, pp. 376 ff.
[4] *Human Knowledge*, p. 505.

that are equally consistent.'[1] But the scientific view seems to be more consistent, and simpler, than any of the suggested alternatives, and for these reasons to be preferred. The postulates, then, cannot be proved, cannot be based upon experience, but by their use we can arrive at scientific laws which are confirmed by experience. Their justification is that they enable us to reach a view of the world which is confirmed, and not denied, by experience, and Russell feels that such a justification, if not all that could be wished for, is at least adequate to make the acceptance of the postulates reasonable.

From the preceding discussion it might appear that Russell believes that events are the ultimate constituents of the world. This view was advanced in several earlier works, including *The Analysis of Matter*, and made the basis of Russell's version of 'neutral monism'. There is nothing in *Human Knowledge* inconsistent with this metaphysics, although Russell does not explicitly discuss it as 'neutral monism'. Yet in *An Inquiry into Meaning and Truth* and *Human Knowledge* Russell suggests a further analysis of 'event' that makes it no longer an ultimate, unanalysable constituent.[2] His new analysis does not introduce any conflict with the results he has found employing events, in fact, generally in *Human Knowledge* his discussion is in terms of events, as mine has been. But now instead of assuming 'event' to be unanalysable, we find that it can be replaced by a complex structure, and consequently his constructions using chains and series of events can be restated in terms of these new complexes. I shall discuss this analysis of 'event' only briefly, since it does not significantly affect Russell's problem as I have been interpreting it.

Both the notion of 'event' and 'particular' should be discarded, Russell believes. The latter implies an ultimate and indefinable numerical diversity, with the difficulty that the particular itself remains unknowable. It is a difficulty comparable to that found in the older conception of 'substance', which

[1] *Human Knowledge*, p. 324.

[2] *Ibid.*, pp. 82 ff., 292 ff; *An Inquiry into Meaning and Truth*, pp. 120 ff., 286 ff.

was something of which various qualities were predicated, and which itself, apart from these qualities, could not be known. Russell maintains that we can avoid these difficulties if we replace the notion of 'particular' in the sense I have discussed with 'particular' considered as a complex of qualities. According to this latter view, qualities may recur at different times in different places.[1] E.g. the quality which is the definite shade of green I see this year in the forest will be the same quality that I may see next year in a painting. In spite, however, of the fact that qualities when they recur are the *same* qualities, it is still possible to have unique 'events'. The shade of green never occurs by itself, but always as one member of a complex of qualities, the other members of which may be shape, size, hardness, and even qualities of being to the left or right (or above or below) some other quality in my visual field. Experienced temporal and spatial qualities are as much qualities as colour or shape. It is then highly improbable that two total complexes of qualities will ever recur, although this recurrence is not logically impossible.[2]

Between qualities, as between events, the relation of 'compresence' can be established, using the same meaning of 'compresence' as I gave earlier in the construction of points and instants. A group of compresent qualities can be found such '(*a*) that all the members of the group are compresent, (*b*) that nothing outside the group is compresent with every member of the group. Such a group I shall call a "complete complex of compresence".'[3] All of the members of this group may occur as members of many other complexes of qualities, but it is this particular group of compresent qualities which Russell finds it highly probable to believe never recurs. Each person's total momentary experience constitutes such a complete complex

[1] Although it might seem that qualities would be universals, Russell denies this. See his discussion in 'Reply to Criticisms', in *The Philosophy of Bertrand Russell*, pp. 685, 714.

[2] *Human Knowledge*, p. 295.

[3] *Ibid.*, p. 294.

of compresence, and will be an 'event' of his experience. It is possible to extend these definitions to events beyond a person's experience, and to the point-instants of the physical world.

The stress on the importance of the principles of inference should not lead us to overlook the conceptions familiar from Russell's earlier writings which are restated at considerable length in *Human Knowledge*. For example, empirical knowledge is distinguished into data and derived beliefs. Beliefs which are data are those which are uninferred, in the sense of 'uninferred' I have discussed in Chapter III, and Russell believes that justifying inferences from percepts to physics is still an important philosophical problem. It is this justification which the postulates are supposed to aid in providing.

The inferences which the postulates are to justify are those which result in what Russell maintains is the picture of the world that science gives us. I have discussed the characteristics which Russell believes this world to possess; and having accepted this picture, Russell invokes the postulates to help validate the inferences from percepts to that world. But what are the reasons for accepting this picture in the first place? The acceptance of scientific laws as far as they have been verified does not commit us to accept Russell's view of the world. It is the investigation to determine which, if any, view of the world is most compatible with science that is one of the problems of a philosophy of science. No one view has as yet been found to be implied by the laws of physics. Russell himself has suggested several alternatives, that the laws of science can be interpreted on the basis of percepts alone, and the view which he now favours. Yet there are other possibilities besides these. He has accepted the view he favours in *Human Knowledge* on such grounds as apparent greater consistency, simplicity, and believability. But Russell's own standards require a sounder justification than such reasons provide; he wishes to show what principles are required such that, given percepts, we can infer this view of the world, but it seems equally a problem on what

principles such a view is justified in the first place. The postulates are of no help in establishing this view of the world, since they have been adopted assuming the truth of Russell's picture of the world. They are needed to justify inferences to a view of the world which in turn is only justified on grounds of simplicity and believability. If we are going to require principles to justify inferences to this view of the world, we should have at least as much justification for the view itself.

It remains to see what purpose constructions can now serve. In the earlier works they had the importance of being able, so Russell maintained, to make it possible for us to dispense with inferences to material objects and physical entities, thus avoiding 'risky' inferences. But as I discussed in connection with their employment in *The Analysis of Matter*, it is doubtful if it can be maintained that constructions containing unperceived events can serve such a purpose. It is no doubt true, as Russell argues, that a construction in terms of a group of events has advantages over a permanent material object, but it is not a difference which makes the inference to one any less remote from percepts than the other. Permanent material objects were originally assumed for the same reasons as Russell assumes groups of events independent of the observer, that is, continuity and objectivity of reference. In Russell's terms, it might be said that permanent material objects preserved the truth of science when science was in an early stage. But as the truth of science which Russell accepts changes, so does the notion of a permanent material object change to that more sophisticated notion of a group of events. The nature of the inferences to either is the same, however, and requires essentially the same justification. At one point logical constructions were supposed to aid in that justification by making unperceived events unnecessary, but now that unperceived events in part compose the construction that construction is of no aid in their justification. In fairness to Russell, it should be noted that he does not emphasize the construction of matter to the extent he did in earlier works, and makes no explicit claim that its construc-

tion does help to validate inferences to unperceived events. Apparently, the purpose which constructions now serve is that they make it possible to consider an 'object' a group of events, both perceived and unperceived, and to make it possible to avoid a separation between mental and physical, thus leading to a metaphysics similar to the 'neutral monism' that I have discussed previously.

The evaluation of the construction of the entities of physics raises questions similar to those I discussed in connection with *The Analysis of Matter*. I mentioned that Russell's view of the 'interpretation' of science in *Human Knowledge* maintained that the interpretation should be in terms of sensible data.[1] He says in discussing absolute time that 'Whenever a body of symbolic propositions which there is reason to accept can be interpreted without inferring such-and-such unobserved entities, the inference from the body of propositions in question to these supposed entities is invalid, since, even if there are no such entities, the body of propositions may be true.'[2] The construction of points and instants (and point-instant) constitutes an interpretation of these terms and presumably would invalidate the inference to the 'unobserved' entities.[3] The constructions do, no doubt, make that inference unnecessary, but the difficulty again is that they employ unperceived events which in turn require the same kind of justification that Russell demands for the rejected 'unobserved' entities. The situation is similar to that of the construction of matter, and the reasons for abandoning the notions of absolute time and space analogous to those for giving up permanent material objects, but these reasons are not the kind that will increase the

[1] *Human Knowledge*, pp. 238, 242.

[2] *Ibid.*, p. 268.

[3] 'If it is arrived at by inference, the inference is of just that kind that I seek to invalidate by the principle of substituting constructions for inferences. The basis of this principle is that, where a suitable construction is possible, this very fact invalidates the inference, since it shows that the supposed inferred entity is not necessary for the interpretation of the propositions of the science in question.' 'Reply to Criticisms', in *The Philosophy of Bertrand Russell*, p. 699.

soundness of the construction. The constructions of points and instants are clearly not interpretations in terms of sensible data, again because of the inclusion of unobserved events. The reservations which I held concerning these constructions in Part III of this Chapter will thus apply to these constructions as presented in *Human Knowledge* as well.

CHAPTER V

CONSTRUCTIONS AS A METHODOLOGICAL PRINCIPLE

PART I

THE 'NEW METHOD' IN PHILOSOPHY

Logical constructions are an essential element in what Russell maintains to be the solution of the problem of the inference to material objects, as they were of central importance in his analysis of descriptions and numbers. They are one of the principal features of what Russell proposes as a 'new method' for philosophy. In some works, for example *Our Knowledge of the External World*, constructions are not only presented as a solution to certain definite problems, but the analysis of the problem itself is supposed to demonstrate the effectiveness of Russell's new method. A new method for philosophy is desirable, Russell believes, to procure for philosophy a progress comparable to that which has been achieved by science. In this Part, I shall discuss the characteristics of this method, determine the extent to which it seems capable of arriving at new and sounder conclusions, and finally, determine if Russell's results are properly illustrative of that method.

A new method for philosophy presumably should meet the tests applicable to any method, and I shall attempt to evaluate Russell's method by such tests. I believe that the usual meaning of 'method' is broad enough to cover two functions: a method of discovery, which can be a new technique of research that uncovers hitherto unknown facts, or a method of proof, which is a technique or procedure for establishing the truth of propositions not so far definitely established. The usual procedures

of deductive and inductive logic could serve as examples o method in this latter sense. The reader might very well expect some justification for this distinction between types of method, but I am introducing it here only as a broad criterion by which Russell's method can be judged. If his method performs either (or both) of these functions, then it could justly be called a method; the further claims which could be made for it would depend upon the extent of its performance. The desire for a method that will produce the sure results and continual progress of science is a recurrent hope in the history of philosophy; but whatever results 'new' methods have had they have achieved nothing so impressive as their authors have claimed. It is interesting to see how far Russell's contribution achieves this aim.[1]

In the Preface to *Our Knowledge of the External World* we find the following statement:

> The following lectures are an attempt to show, by means of examples, the nature, capacity, and limitations of the logical-analytic method in philosophy. This method . . . has gradually, in the course of actual research, increasingly forced itself upon me as something perfectly definite, capable of embodiment in maxims, and adequate, in all branches of philosophy, to yield whatever objective scientific knowledge it is possible to obtain (v).

The distinguishing feature of this method is that it gives logic central importance, an importance so great that Russell entitles one of his lectures in *Our Knowledge of the External World* 'Logic as the Essence of Philosophy'.

> . . . the topics we shall discuss [especially those of the basis of our knowledge of the external world and of the entities of physics] all reduce themselves, in so far as they are genuinely philosophical, to problems of logic. This is not due to any accident, but to the fact that every philosophical problem, when it is subjected to the neces-

[1] On the general subject treated in this Part, see *Our Knowledge of the External World*, Chapter II; *Mysticism and Logic*, Chapter VI; Morris Weitz, in *The Philosophy of Bertrand Russell*, pp. 110 ff. An excellent, fuller treatment of Russell's method is C. J. Ducasse, *Philosophy as a Science*, New York, 1941, Chapter V.

sary analysis and purification, is found either to be not really philosophical at all, or else to be, in the sense in which we are using the word, logical.[1]

It is not traditional logic which Russell considers important here, but symbolic logic; Russell modestly does not claim to discover a method which could have been known to the Greeks, but rather a method that could only be devised following the modern developments in logic.

It should be noted that Russell says 'in the sense in which we are using the word, logical', for it turns out that he is using 'logic' in a broader sense than is customary; overlooking this qualification can lead to a misleading view of his method. Logic, as Russell is using the term, is composed of two parts, the first of which only is normally called 'logic':

> On the one hand [logic] is concerned with those general statements which can be made concerning everything without mentioning any one thing or predicate or relation. . . . On the other hand, it is concerned with the analysis and enumeration of logical *forms*, i.e. with the kinds of propositions that may occur, with the various types of facts, and with the classification of the constituents of facts. In this way logic provides an inventory of possibilities, a repertory of abstractly tenable hypotheses.[2]

The first part of logic would correspond to a formal deductive system, e.g. that of *Principia Mathematica*; the second part would include an investigation of the meaning of logical entities, of the correspondence of propositions with facts, and an analysis of the entities and relations in the real world which correspond to those of the logical system.

The beliefs which Russell included in the second part of logic at the time of writing the preceding quotation are those I presented in Chapter III in discussing logical atomism. We found there that Russell maintained that there was a hierarchy of

[1] *Our Knowledge of the External World*, p. 33.

[2] *Mysticism and Logic*, p. 112; see *Our Knowledge of the External World*, pp. 50 ff.

propositions which corresponded to one of facts, that there was a correspondence between the constituents of the facts and of the propositions which expressed them, and it was the presence or absence of this correspondence that determined the truth or falsity of propositions. I discussed at that time the technical meaning which Russell gave to 'fact'. The task of philosophy was then to find the terms of facts corresponding to constituents of propositions which we were investigating, e.g. those concerning material objects, matter, time, space. However, Russell's views concerning this second part of 'logic' were by no means unchanged. At the time he wrote *Principles of Mathematics*, he did not even describe his corresponding views as part of 'logic'. Yet as I discussed in Chapter II, Russell investigated problems similar to those that later he included in the second part of logic. In fact, there is a curious similarity between the logical doctrines of the two periods, keeping in mind that in the earlier work, the use of 'sentence' is analogous to that of 'proposition' in the logical atomism period, while the earlier use of 'proposition' is analogous to the later one of 'fact'. But his earlier metaphysics was considerably more realistic than that of logical atomism; the world was composed of an infinite number of entities, including physical objects and logical entities such as concepts and relations. According to his early view, 'sentences' corresponded to 'propositions', the latter were composed of subjects, or 'terms', and 'concepts' which could be either verbs or adjectives (predicates). The terms of a proposition were among the real entities of the world, either existent or subsistent, such as a physical object, or a particular universal; the concepts were subsistent entities. A sentence referred to its corresponding proposition since the constituents of the sentence denoted or 'meant' the corresponding constituents of the proposition. An analysis of the meaning of a sentence becomes a determination of the constituents of the proposition denoted by the constituents of the sentence, and these constituents will be real entities of some kind.

The second part of logic is not then a relatively stable body of beliefs such as compose the first part. This second part is, in fact, more a philosophical view of the world than it is a method for discovering that view. In short, it is the result of some method, rather than a method itself. Russell's view of the doctrines which comprise the second part of 'logic' depend upon various metaphysical assumptions about the nature of the world. If metaphysical assumptions are to be included in the new 'method', there would seem to be many metaphysics with equally valid claims for inclusion. The claims which Russell makes in favour of his metaphysics are, as far as I can see, that it gives an intelligible picture of the world, which are the least grounds for which belief in any metaphysics is claimed. But it is the very truth of philosophical doctrines, including metaphysics, which Russell maintains his method will settle, and to include some of these doctrines in that method makes the method to that extent question-begging.

Accepting for the moment this second part of logic as a part of the 'new method', what is the significance of Russell's statement that 'every philosophical problem . . . is found . . . to be . . . logical'? If Russell were restricting 'logic' to its more normal use, it would dismiss all problems except those arising in the examination of a formal system; but his broader use of the term results in no such drastic action. In this broader sense, problems of logic include the classification of facts, of propositions, the determination of the correspondence between propositions and facts, and the determination of the constituents of the facts which correspond to given propositions. For example, the problem of the existence of material objects is a 'logical' problem, since it requires a determination of the constituents of the facts that correspond to propositions concerning material objects. It will end either in finding a constituent that does correspond to words for material objects, or if none such can be found, in analysing the original proposition in some other manner. The latter, of course, is what Russell proceeded to do in his construction of material objects, and similarly in

the construction of the entities of physics. Thus although presumably we are discussing 'logical' problems they are the familiar problems of ordinary philosophy.

One is apt to alarm himself unduly if in a hasty reading of the title 'logic as the essence of philosophy' he assumes that Russell intends a wholesale dismissal of all philosophical problems aside from those usually called 'logical'; actually the problems investigated by Russell's new method are the usual problems of philosophy. It will be apparent from a hurried reading of the table of contents of Russell's works that he does in fact consider many of the usual problems of philosophy, such as the nature of material objects, of mind, of space and time, and others which I have not treated. The investigation of all of these problems can reduce to the analysis of the facts and propositions involved in them, which is 'logic' in Russell's broad sense of the word. We are not then going to meet a radical reform of the subject-matter of philosophy, but a 'new' method for the investigation of that subject-matter.

One claim which Russell makes for his method is that it is 'perfectly definite, capable of embodiment in maxims'. This statement suggests that the method can be briefly summed up in a few 'maxims' that can then be applied in order to a given problem and result in finding as adequate an answer to the problem as we can hope to obtain. Unfortunately, there is only one such 'maxim' given by Russell, 'wherever possible, logical constructions are to be substituted for inferred entities'.[1] It is not clear that Russell does in fact maintain this to be one of the maxims of his method, it is not explicitly discussed as such; but the use of the word 'maxim' as characterizing the quotation, and the fact that he uses it as one of the central features of his analyses, leads to the assumption that he intends it to be one of those maxims. I shall discuss the question of the nature of logical constructions shortly, so I shall not discuss the adequacy of this maxim now. What we are left with so far as constituting the discussion of the 'new method' are Russell's general

[1] *Mysticism and Logic*, p. 155.

statements, such as the quotations already given, his discussions of the content of logic, and in addition, this one maxim.

It is unfortunate that Russell does not formulate precisely what are the steps in the application of his method; for in the absence of such a discussion we can evaluate that method only by an examination of the results which Russell obtains, presumably by its use. It is apparent that he has made use of constructions in his investigations. He has also made use of the theories contained in the second part of 'logic'. The difficulty with these theories is that they are primarily metaphysical, and Russell has provided no criteria to determine the certainty of the beliefs contained in these theories. It will be necessary, before we can advocate his method, to distinguish the methodological from the metaphysical elements, and to determine the part played by the strictly methodological aspects. Considering the statements I have cited so far, the only element in his method that is new to any extent is his use of constructions.

Further statements of Russell's concerning his method clear up some of the difficulties I have discussed so far, but at the same time raise new problems by conflicting with statements I have previously discussed. Thus, he maintains that the important role of logic in philosophy is that its results are useful in the investigation of other types of problems. It is, he says, 'a suggester of new possibilities'. According to traditional logic only certain types of propositions were possible, and particular philosophical problems had to conform to these types. Modern logic increases the possible kinds of propositions, and it may be that after further examination these problems will fit more correctly into one of the new forms. This point is illustrated by Russell's own analysis of space.[1] It is found that the study of order in symbolic logic (and mathematics) includes the kind of serial relation which is expressed by 'between', and that this relation is analogous to 'between' as it occurs in space empirically given. An analysis of space in terms of order would have been difficult, if not impossible, Russell maintains, when the

[1] *Mysticism and Logic*, pp. 113 ff.; *Philosophy*, pp. 296 ff.

only kind of propositions known were subject-predicate ones, since one cannot analyse '*a* is between *b* and *c*' into a subject and predicate.[1] The new logic, however, includes propositions like those referring to 'between' among even more general classes of relational propositions, and thus makes possible an analysis of problems that could not successfully be carried out employing only traditional logic.

This claim for logic seems clearly a valid one; the usefulness of logic in this connection can be seen by the importance of the knowledge obtained by recent logical and mathematical studies. But it is a claim of a different nature from the claim I have previously discussed of the identification of philosophical problems with logical, of logic being the *method* of philosophy.

> Logic, instead of being, as formerly, a bar to possibilities, has become the great liberator of the imagination, presenting innumerable alternatives which are closed to unreflective common sense, and leaving to experience the task of deciding, where decision is possible, between the many worlds which logic offers for our choice.[2]

According to this statement, logic is not the method for deciding between the new possibilities which it suggests; logic will suggest several possible analyses of a problem, and we must then determine which is correct by 'experience', by, presumably, the usual methods of investigation and thinking.

A more general, and also somewhat weaker, form of Russell's method than that given in the statements I discussed at the beginning of this Part, considers his method simply the 'logical-analytic' method, without claiming that it can be formulated in maxims or identified with logic. This, in fact, is the way we find Russell usually characterizing his method in later writings. The 'logical-analytic' method is now that method which analyses philosophical problems by using the results of modern logic, although 'logic' is still interpreted to include the type of beliefs which I referred to as the 'second part' of logic. He com-

[1] *Principles of Mathematics*, pp. 95, 221 ff.

[2] *Philosophy*, p. 297.

pares this method to a tool comparable in power and function for philosophy to that of mathematics for physics;[1] he maintains that we can break up large philosophical problems into smaller ones, attack these smaller ones, find a successful solution, then by building upon the cumulative results of the small problems reach a solution of the original larger one.

> Many matters which, when I was young, baffled me by the vagueness of all that had been said about them, are now amenable to an exact technique, which makes possible the kind of progress that is customary in science. Where definite knowledge is unobtainable, it is sometimes possible to prove that it is unattainable, and it is usually possible to formulate a variety of exact hypotheses, all compatible with the existing evidence. Those philosophers who have adopted the methods derived from logical analysis can argue with each other, not in the old aimless way, but co-operatively, so that both sides can concur as to the outcome.[2]

In discussing this method, I should like to see first whether in fact it is unique of this new method to break up large problems into smaller ones and find definite answers to the smaller before attacking the larger. If this is interpreted strictly as a methodological principle, it seems to me that this refers to the aim that any theory should account for as many relevant factors and conditions as possible, and that as many facts as possible should be used to verify the theory. In short, in forming generalizations one cannot afford to overlook relevant details. Interpreted in this way, it is the customary procedure for any constructive thinking, and not the private possession of the method of analysis. If, however, it is interpreted as a characteristic of an adequate philosophical system, and I believe Russell does so interpet it, then it means that this system is one in which such problems as the nature of mind and matter are solved by reducing them to the actual particulars and universals involved, rather than by the answer given by, e.g., speculative idealist philosophies. Following this interpretation, the principle that

[1] *Our Knowledge of the External World*, p. 239.

[2] 'My Mental Development', in *The Philosophy of Bertrand Russell*, p. 20. See *A History of Western Philosophy*, p. 834.

large problems should be broken up into smaller ones, seems hardly more a *methodological* principle than the second part of logic was methodological. It depends upon certain metaphysical views of the world, that the world is such that philosophical problems are solved when they are broken down into definite particulars and universals. This view may or may not be justified, but it is its very justification which is at issue. Thus it may very well be a principle of Russell's philosophy to break up large problems, but it is hardly a principle of method which should be universally adopted until we have determined that Russell's philosophical position is the true position.

This latter point I have raised is the essential issue in what might be called the 'justification' of analysis. There are two reasons why such justification should be discussed, only the first of which is recognized by Russell. (1) It has been maintained, especially by the idealist position, that analysis is 'falsification'. Analysis assumes one can arrive at the correct meaning of a complex whole by analysing it into its component parts. If, however, the whole has a meaning which cannot be obtained by simply reconstituting it from its parts, then analysis has lost some of the meaning and has 'falsified' the whole. Thus ascertaining the parts of the whole would give us some information, but could not take the place of an examination of the complex as a whole. Thus Russell says:

> . . . I share the common-sense belief that there are many separate things; I do not regard the apparent multiplicity of the world as consisting merely in phases and unreal divisions of a single indivisible Reality. It results from that that a considerable part of what one would have to do to justify the sort of philosophy I wish to advocate would consist in justifying the process of analysis.[1]

Russell himself, in *Principles of Mathematics*, although he had not at that time explicitly formulated his analytical method, was not certain that analysis was not falsification. Propositions, he believed, were 'unities', and unities were wholes that were not completely specified when all the parts were known. 'A pro-

[1] *Monist*, 1918, 496.

position has a certain indefinable unity, in virtue of which it is an assertion; and this is so completely lost by analysis that no enumeration of constituents will restore it, even though itself be mentioned as a constituent.'[1] But even here: 'In short, though analysis gives us the truth, and nothing but the truth, yet it can never give us the whole truth. This is the only sense in which the doctrine [that analysis is falsification] is to be accepted. In any wider sense, it becomes merely a cloak for laziness, by giving an excuse to those who dislike the labour of analysis.'[2] Aside from propositions, there was no restriction on analysis, applied to other kinds of complexes than propositions it was perfectly valid. Later, at least in the logical analytic period, the qualifications on the use of analysis on propositions were removed.[3]

(2) The foregoing 'justification' of analysis has been concerned with showing that there are wholes which can be analysed without falsification. But this carries with it certain assumptions about the subject-matter that is being analysed. 'Analysis' as it is used by Russell, and also by many other contemporary British philosophers, refers to a procedure carrying with it certain assumptions as to the particular material that is analysed. This can be seen, e.g., in two views of analysis presented in the *Proceedings of the Aristotelian Society* for 1932–3. Miss Stebbing, after distinguishing grammatical analysis (analysis of the combinations of words in a sentence), and metaphysical analysis, goes on to discuss metaphysical analysis, which to her is the important kind:

> The aim of metaphysical analysis is to determine the elements and the mode of combination of those elements to which reference is made when any given true assertion is made. . . . Starting from a given proposition a progressive attempt is made to indicate sets of elements arranged in a certain way which is such that it is true *both* that the proposition refers to each member of the set *and* that

[1] *Principles of Mathematics*, p. 466.

[2] *Ibid.*, p. 141.

[3] See also *Philosophy*, pp. 247–8. A somewhat contrasting, more complex view of analysis is given in *An Inquiry into Meaning and Truth*, pp. 411 ff.

no member of the set involves any further arrangement of elements.[1]

Configurations of elements make up 'facts', and facts are the ultimate reference of all true statements. 'To know what precisely there is in the world is to know what are the facts which together make up, i.e. constitute the world.'[2] Metaphysics is the study of the structure of the facts in the world, and metaphysical analysis determines the elements in the facts referred to by any given statement. However, Max Black maintains that 'logical analysis is a method for elucidating the structure of systems of symbols or "languages", i.e. any set of symbols used in recurrent combinations for communication between persons. . . . What is meant by saying that language has a structure is essentially that certain elements . . . recur as members in various complexes of elements while remaining recognizably the same in these different contexts, so that complexes can be transformed into one another by the reciprocal exchange of elements.'[3] Analysis then makes these complexities explicit by discovering laws for transforming symbols and by the manufacture of new symbols of sufficient visible complexity.

Russell himself believes that analysis is of reality; as I have mentioned, in *Principles of Mathematics* he maintains that analysis of a problem will reveal various propositions, these propositions in turn will consist of real entities, either terms or concepts. In the logical atomism period, he believed analysis would disclose certain objective 'facts', with a definite structure and real constituents. In both cases it is assumed that the propositions or facts of logic are real entities with particular enumerated properties; analysis gives us knowledge of objects and circumstances concerning the objective world. It seems then clear to me that analysis as Russell conceives of it is based on a

[1] L. S. Stebbing, 'The Method of Analysis in Metaphysics', *Proceedings Aristotelian Society*, N. S. XXXIII (1932–3), 79.

[2] *Ibid.*, p. 65.

[3] Max Black, 'Philosophical Analysis', *Proceedings Aristotelian Society*, N. S. XXXIII (1932–3), 238–40.

metaphysical view of the world, rather than being a procedure for discovering that view. Russell maintains that the results he achieves justify 'analysis',[1] but if it is justified, it is as a metaphysics and not as a method.

The 'justification' of analysis, or of one type of analysis rather than another, is a justification of a metaphysical view; accepting a view as to the nature of what is analysed does not show that a particular 'method' is required for that analysis. In the examples I have given, I cannot find any strictly methodological reference, any new principle of investigation or proof involved in the application of these metaphysical views to a given philosophical problem. The result is that a significant part of what Russell maintains is his new method, 'analysis', or what at one time comprised the 'second part' of logic, are rather part of his metaphysics than his method. My view is not only borne out, I believe, by an examination of Russell's explicit statements concerning his method, but by the absence of any evidence to the contrary gained from a consideration of the solutions he advances to various problems. There remain still, however, two of Russell's methodological claims that are valid, that is, the employment of modern logic as a 'suggester of new possibilities', and the use of logical constructions. Both of these are, I believe, legitimate methodological principles in that they contribute to the formation of hypotheses and definitions which without their use would be difficult, if not impossible. The use of these two principles is an important and significant contribution, but it is unfortunate that Russell has made claims far exceeding the use of these two principles. His further claims, I believe, are not valid, and the controversy caused by his extreme claims is apt to overshadow the very real importance of what he has contributed to modern methodological technique.

[1] *Monist*, 1919, 365–6.

PART II

LOGICAL CONSTRUCTIONS AND INCOMPLETE SYMBOLS

The employment of logical constructions has been of great effectiveness in enabling Russell to advance solutions for the philosophical problems he investigates, and is one of the distinguishing methodological features of his work. It is important to attempt a characterization of the nature of logical constructions. They have been used throughout Russell's work in solving problems of different kinds, the logical and mathematical problems, and the problems of material objects and the entities of physics, but so far I have not discussed the common features of constructions in all of these applications. It is not easy to obtain any clear picture of their nature, in part because Russell's own discussion is not always clear; but after discussing certain misconceptions which may arise because of the terms Russell sometimes uses in describing constructions, I shall try to find certain of their essential features.

The term 'logical construction' has the advantage of being vivid and forceful, but can also, unfortunately, be misleading. A 'construction' is normally something actually built by some rearrangement of material, for example, a house; but in Russell's 'constructions' no such building takes place. In the normal sense of the word they are not constructions at all. There is no new object produced as a result of the construction, in fact, just the reverse is the case. The material used in the 'construction', e.g. particulars, has previously existed, and continues unchanged, but is merely classified in a new and distinctive manner. In the case of real numbers, not even a new classification is made, the series of numbers has been already acknowledged and utilized. The purpose which Russell maintains constructions fulfil is to show that all the functions of a supposed unperceivable entity can be served by a class of entities whose existence is more solidly grounded.

The class of entities which comprise the construction Russell sometimes speaks of as 'known' entities, while the supposed unperceivable entity is an 'unknown' entity. What he means, as I have discussed before, is that in constructions such as that of number, or descriptions, the entities used in the construction are ones of whose existence and properties we are fairly certain (although 'class' later was found in turn to be analysable as a construction); while the supposed entity whose existence the construction is expected to make unnecessary is 'unknown', because it is not directly experienced. In the constructions of material objects and physical entities the 'known' entities were particulars with which we are acquainted, or whose existence is known without inference, and the 'unknown' entities known only by inference. In these latter constructions Russell was not successful in employing only 'known' entities, and his speaking of constructions as using only 'known' entities may seem questionable. Even so, the terms 'known' and 'unknown' are not a clear characterization of what is employed in a construction; they are meaningful only when Russell's further epistemological distinctions are kept in mind, and to avoid confusion might better not be used at all.

In this book I have tried to avoid saying that the construction *replaces* the unknown or inferred entity. The reason for this is that numbers, descriptions, and material objects, for example, are 'incomplete symbols'. It will be remembered that an 'incomplete symbol' has no meaning in isolation, and can be defined only in certain contexts. Symbols which do have meaning in isolation, e.g. proper names, have meaning, Russell maintains, because there exists some object for which they can stand. Thus, if material objects were entities, e.g. if 'barn' referred to an entity, words such as 'barn' would denote these entities and would have meaning in isolation. Russell maintains, however, that these entities cannot be known, or validly inferred, and that his analysis reveals that it is possible to consider 'barn' an incomplete symbol. It would also not be correct to say that 'barn' (in isolation) denoted a class of appearances or

events, for classes have been defined by Russell as incomplete symbols, hence not themselves entities. 'Barn' is then defined 'in use', and entire statements in which the word occurs are analysed. For example, 'there is a red barn on that hill' will be found, employing the procedure of constructions, to be equivalent to a statement which refers to various classes of particulars, but it will not be possible to single out some one group as corresponding to the word 'barn'.

If such things as numbers, descriptions, material objects are incomplete symbols, in what sense can we legitimately speak of 'substituting' logical constructions for inferred entities? In one of his versions of 'Occam's Razor', Russell states 'logical constructions are to be substituted for inferred entities'.[1] This statement gives the impression either that constructions are objects which can be substituted for other objects, or that constructions are symbols which can be substituted for symbols representing inferred entities. The first is clearly not what Russell intends, and the second, interpreting 'substitute' in the sense in which symbols are 'substituted' for each other in a logical system, is also not the case, since the symbol apparently denoting an unknown entity is really an incomplete symbol. When a symbol is incomplete it cannot simply be removed from one expression and substituted in another, since its meaning depends upon the whole expression in which it occurs. What Russell probably means by 'substitute' can be seen better from another statement he gives of 'Occam's Razor': 'substitute constructions out of known entities *for inferences to* unknown entities'[2] (my italics). Instead of inferring the existence of material objects as entities, we can 'substitute' statements concerning the establishment of correlated groups of particulars. We must be careful in interpreting statements in which Russell speaks of 'substituting', or 'replacing', entities by constructions that the real nature of a construction be kept in mind, and avoid attributing too literal a meaning to 'substitute'.

[1] *Mysticism and Logic*, p. 155.
[2] *Monist*, 1919, 362–3.

One finds often that Russell refers to incomplete symbols, such as descriptions, material objects and the like, as logical 'fictions'.[1] The use of the word 'fiction' in this context seems even more dubious than the use of some of the other words I have been discussing in this Part. 'Fiction' can convey the impression that, for example, objects described, and material objects, do not exist as entities; and such an impression does conform to Russell's view. Although he does not actually deny the existence of these peculiar kinds of entities, yet he finds no reason for believing in that existence. The word 'fiction' might imply that Russell believes these entities do not have any real existence, but are rather creations of the mind, something actually put together by the imagination, but having no further existence. It seems clear that Russell does not intend to maintain such a view as this.[2] The interpretation of 'logical fiction' which conforms closest to Russell's view is that an incomplete symbol is a 'fiction' because at first sight like other symbols it appears to denote some entity, but actually, after analysis, we find that this is not the case. They are symbols which have a certain convenience in language and logic, but are convenient symbolic devices only; they do not denote any constituent of the world. Whether or not such an interpretation, if correct, justifies the use of the word 'fiction', I shall not attempt to decide.

It may now be possible to determine more precisely what a logical construction *is*, after clarifying possible misapprehensions associated with the term. My discussion shall continue to be general, since a more thorough analysis requires the detailed consideration of technical logical problems, especially that of the nature of incomplete symbols. First, a construction is not a *thing*; it is not an entity actually composed of particulars, since this is one type of entity that constructions are designed

[1] See, e.g., *Introduction to Mathematical Philosophy*, p. 182; *Monist*, 1919, 222.

[2] See, on this general point, L. S. Stebbing, 'Constructions', *Proceedings Aristotelian Society*, N. S. XXXIV (1933–4).

to make unnecessary. It is not identical with a class of particulars, since this class in turn is an incomplete symbol, and defined by means of a construction. If constructions were things, they might be said to be denoted by incomplete symbols; but this is impossible, since incomplete symbols denote nothing. A construction is not identical with an incomplete symbol; for, if that were the case, the word 'barn', or the symbol '2' would be logical constructions. But giving the symbols we wish to analyse another name does not facilitate progress; logical constructions are presumed to provide an analysis of these symbols, and consequently are not those symbols themselves. What a construction is, essentially, is rather an actual procedure of analysis. A logical construction is an analysis of a term which shows that the term is, in fact, an incomplete symbol and can be given a definition in use. A construction, however, is not only a symbolic definition, it at the same time, to Russell, represents the analysis of a real state of affairs, either metaphysical or epistemological. My conception of 'construction' accords with the earlier discussion in this Part, and also, I believe, accords with what Russell is actually doing in his analyses that I have presented.

Frequently, however, we find Russell speaking as if numbers, and material objects, for example, *are* constructions, in the sense that they are entities constructed out of other entities. Such a usage, as can be seen if the foregoing is accepted, is not correct; but it is a convenient way of speaking, and is harmless if it is realized that it is only a brief manner of expression. It is, in fact, one that for the sake of simplicity I follow myself at times in this book. Thus, saying 'a material object is a logical construction' is only a briefer and less clumsy, and also, of course, less precise way of saying 'statements concerning material objects are equivalent to statements concerning classes of particulars'. In this last statement the fact that 'material object' is an incomplete symbol is taken into account, while in the former statement it would appear that such symbols are not incomplete.

METHODOLOGICAL PRINCIPLE

Constructions are a methodological procedure, and one of the important features of Russell's method. The question still remains of the validity of Russell's 'maxim', that constructions should be employed 'wherever possible'; and also whether Russell's actual use of constructions is an illustration of such a maxim. One difficulty with evaluating the maxim is that constructions may be employed for different purposes. They can be used in a metaphysical sense, in attempting to dispense with entities which otherwise it is believed would be part of the metaphysical structure of the world; they can be used in what I have been calling an 'epistemological' sense, in analysing whether beliefs in material objects can be 'certain'. Russell also in his later writings used them in a sense I have not discussed here, in which they are used to define certain words in terms of others, these others being a 'minimum vocabulary'.[1] In which of these uses of constructions are we supposed to apply the maxim? We could apply it in all, or only in one and not the others. The statement which Russell gives of his maxim in *Mysticism and Logic* implies a metaphysical use of it, but that in *Philosophy of Logical Atomism* (see above, p. 216) could imply either or both of the first two. There are, however, further difficulties with the maxim which makes it unnecessary to settle this issue.

The maxim that Russell advocates he states in one place as: 'Wherever possible, logical constructions are to be substituted for inferred entities.'[2] The aim, apparently, is to employ as few basic entities as possible, reducing all others to these by 'constructions', by using definitions in use which refer only to the basic entities. This will serve Russell's metaphysical purpose, and I believe also his epistemological. The difficulty with the maxim is in the word 'possible'. If we are going to adopt this as a methodological principle, we should be able to determine

[1] 'My Mental Development', in *The Philosophy of Bertrand Russell*, p. 14. The notion of a 'minimum vocabulary' is discussed at length in *An Inquiry into Meaning and Truth* and *Human Knowledge*.

[2] *Mysticism and Logic*, p. 155.

what entities can be constructed, what entities cannot be so constructed. Russell does not give criteria which will enable us to determine whether something can be a construction or not, unless one assume that such a statement as 'What is logically convenient is likely to be artificial'[1] is intended as one such criterion. But this would certainly not be adequate. What is more likely his intention is that the only basic non-constructed entities would be particulars (or events),[2] and that we should attempt to reduce all other entities by means of constructions to these basic entities. If this is the meaning Russell wishes to give the maxim, his discussion should be expanded and illustrated by being carried out as a systematic procedure. For example, beginning from particulars, further entities could be introduced as constructions, and in turn these used to introduce still more entities as constructions, giving rules and examples of the procedure being followed. It would also be necessary to know what entities could *not* be introduced as constructions, if it is found that there are any such, and in this way determine what precisely are the limits to the employment of constructions. The most thoroughgoing attempt at actually carrying out a procedure like that I have just described, in which all other entities besides basic entities are introduced as constructions, is, I believe, to be found in Carnap's *Logische Aufbau*.[3]

If we are to view the employment of constructions as a general rule, we require a more systematic investigation and application of constructions than Russell has given. He has employed constructions in only a few instances, most of which I have mentioned in this book, and has not engaged in the syste-

[1] *The Analysis of Matter*, p. 290.

[2] It is not clear to what extent universals are taken as basic non-constructed data. In some writings all universals seem equally to be known without inference, by 'acquaintance', in others many universals can be 'constructed' from others. See *Principia Mathematica*, I, Appendix C, in which 'similarity' is taken as a non-constructed universal; and Gödel's essay in *The Philosophy of Bertrand Russell*, pp. 141–2.

[3] R. Carnap, *Der Logische Aufbau der Welt*, Berlin, 1928.

matic methodological investigation of constructions that would result from the kind of study I mentioned in the preceding paragraph. Russell himself seems to recognize this, for in his later works the use of constructions is not mentioned as a maxim, but merely as a device which he has found helpful.[1] This does not disparage the importance of constructions, but finding them useful is quite different from giving them a definite status as a generalized methodological procedure.

The fact that Russell has not formulated a systematic procedure for the application of constructions is illustrated by their gradual introduction into his own work. First he found it possible to define numbers, descriptions, and classes by the use of constructions, then later material objects, minds, and the entities of physics. Each construction seems to be a 'discovery', the result of a fortunate accident in determining that some problem can be analysed by means of constructions, rather than the result of following some definite rule of analysis. This is no disparagement of Russell's work, since it might very well be necessary to find that certain problems can be treated by constructions before the notion of a systematic procedure would suggest itself. On the basis of the examples he provides, a definite procedure for the employment of constructions might very well be formulated, but this is a task to which Russell has not chosen to devote himself.

PART III

CONCLUSION

The preceding Chapters of this book have presented Russell's philosophy as illustrating the use of constructions first, as a general methodological principle in philosophy, and second, as employed in analysing the particular problems involved

[1] 'My Mental Development', in *The Philosophy of Bertrand Russell*, p. 14.

in the justification of the belief in the external world. The application of constructions described in Chapter II to mathematical and logical problems was so successful that Russell was encouraged to apply them to many further philosophical problems and make them the central feature of what he proposed as a 'new method' for philosophy. It was to the problem of the basis of our knowledge of the external world that he applied his 'new method' most extensively with the results I have presented in the preceding Chapters. But his 'new method', as I have maintained, is not 'new', nor are the results he has obtained dependent upon such a 'new method'. Russell's conception of analysis depends largely on prior assumptions as to what it is that is being analysed; the process by which the analysis is carried out differs little from the manner in which any philosophical investigation is conducted. The one methodological principle which Russell's work does illustrate is that of the use of constructions, but it is an employment in particular problems in which Russell has been interested; and not a systematic employment governed by rules determining when constructions can be used, when they cannot, and in what manner problems should be attacked so that by means of constructions we can obtain a solution. I find it more important, then, to evaluate Russell's work not as the successive application of a new method in philosophy, but as the analysis of various important and relatively independent problems. I shall not again discuss the logical and mathematical problems of Chapter II, but re-examine the results of his investigation of the problems raised by the analysis of our knowledge of material objects and of science.

The analysis of empirical knowledge, according to Russell, shows that we can divide that knowledge into that which is either basic, in the sense of being non-inferred, or that which is based upon inferences from basic knowledge. The problem of our knowledge of material objects and of science resolves into an examination of the inferences from what we find to be basic knowledge to knowledge of material objects or scientific

knowledge to determine in what way those inferences can be avoided, or (in *Human Knowledge*) justified. Russell feels his general distinction between inferred and non-inferred beliefs is justified since, stating his case crudely, our empirical knowledge must have some foundation, and that foundation must be the individual's own view of the world. This view of the world, to Russell, reduces to the individual's momentary perceptual experience; further extensions to his knowledge which link his own experience together and place it in a larger context of other persons' experience are obtained through inference.

I have criticized several points concerning Russell's proposed substitution of logical constructions for inferences to material objects and entities of science, primarily that such substitution does not really avoid or justify the inferences in question. His constructions all to some extent employ beliefs of the type he feels it is necessary to justify, beliefs in the percepts of others or in unperceived events. Even if by using constructions Russell could solve his problem, it is questionable whether statements obtained by constructions are really equivalent to those referring to material objects. But I have not been too concerned with this question since the prior consideration is whether constructions perform the task that Russell sets for them, and this I believe they fail to do. If we add to Russell's problem as I have stated it the desire to justify the inferences to a metaphysical view of the world, whether that of *Our Knowledge of the External World* or 'neutral monism', the use of constructions is still inadequate for the reasons that it was when primarily epistemological considerations were involved.

Instead of accepting the negative conclusions implied by my criticism, it might be more valuable to re-examine briefly Russell's analysis to see at what point it could be modified to avoid the unsuccessful result at which I maintain he has arrived. First of all, there are some philosophers who would deny Russell's distinction between data and derived beliefs entirely. I have discussed this point briefly before. The difficulties that

Russell has had in distinguishing clearly between what is a datum and what is a derived belief, and in trying to determine in certain specific cases which category a given belief belongs in, seem to lend support to the view that denied the distinction between them. If there were no distinction, our knowledge would form an interconnected whole in which every part was supported by some other, and we could not single out any parts as being epistemologically basic. However, in spite of this, I feel that his distinction can be justified, on essentially the same grounds as he has given. I shall, at least, assume that it can, since otherwise no further examination of Russell's work would be called for. But the acceptance of his distinction in the most general sense, that is, that the individual's own perceptual experience is the basis for empirical knowledge, does not imply that Russell's determination of what type of beliefs constitute basic data is the most adequate one that could be found.

The characterization of data which I believe Russell intends is primarily that it is non-inferred. It is only, he maintains, the momentary sensible experience of the individual that is data by this standard; it is data because that experience constitutes all the evidence required to justify the belief.[1] Any belief of the individual beyond such momentary experience is not data because it requires additional evidence for its justification, and hence is inferred. It has been difficult for Russell to determine precisely where the dividing line between data and inferred beliefs falls, in fact, at times he believes that the notion of data is only a 'limiting conception', something that apparently we can never find. Russell's analysis of data is, I believe, at fault, and is so carried out that we could never arrive at hard data by its means.

Accepting his criteria of hard data, I shall retrace the steps by which Russell arrives at the type of beliefs that he feels most nearly approximate hard data. The simplest approach to my discussion is to repeat a quotation from *Philosophy*:

[1] I am excluding memory from my discussion, as I have in the remainder of this book.

> The actual datum, in each case, is unimpeachable, but the extensions which we make instinctively are questionable. . . , No words exist for describing the actual occurrence in all its particularity; all words, even proper names, are general, with the possible exception of 'this', which is ambiguous. When you translate the occurrence into words, you are making generalizations and inferences, just as you are when you say 'there is a chair' . . . what is really a datum is unutterable, and what can be put into words involves inferences which may be mistaken (12–13).

As I have already mentioned, beliefs such as 'I see a brown cow' are uncertain because of the indefinite amount of evidence required to confirm them. Such beliefs purport to be about material objects, and all beliefs of that type require, Russell maintains, a large amount of confirmatory evidence. Beliefs such as 'I see a brown patch' would also be uncertain, but less uncertain than those about material objects, since less confirmatory evidence would be required. If we were to confine ourselves to stating 'brown patch occurring now', this belief would be still more certain, since we have eliminated reference to the self, which is not known, says Russell, without inference. Such a series of beliefs with more and more restricted reference illustrates the manner in which we can approach to those beliefs which are hard data. But the difficulty with approaching hard data through the analysis of beliefs expressed in this manner is that such expression always involves some objective reference and we can never arrive at really hard data. Even the statement 'brown patch occurring now' assumes that the experience one is having is what others refer to as 'brown', and consequently involves the determination whether or not the patch really is brown. Since such a determination might be false, and cannot be decided merely on the basis of the present sensible experience, the belief cannot be classed as hard data.

Let us consider further the example of an individual seeing something brown, and expressing his belief as 'this is brown'. I shall overlook the complexities in finding the best form of words to express this belief, but my formulation is similar to many of Russell's examples. Normally, I maintain, one intends such a

statement to be one that is either true or false, and usually would not be expressed unless one believed it to be true. This is an example of about as certain a belief as Russell finds, since it seems to refer only to the subject's sensible experience. It is still, however, uncertain by Russell's own standards. In maintaining 'this is brown' is true, one maintains that his sensible experience is similar to others when they say 'this is brown', and the evidence required for determining its truth is comparison with the reports of others when all are observing the same object, possibly some physical standard such as light of a definite wave length. This is clearly more evidence than that provided by the subject's present experience, and hence the belief is an inferred belief. The beliefs which Russell examines are expressed in the manner of my example, and consequently it is impossible to find any really hard data among them. Russell is driven, at least in the selection I quoted in the preceding paragraph, to find the bare unutterable datum itself as the only hard datum, but we are then faced with the difficulties of finding beliefs which require only this datum as evidence for their truth.[1]

It might, however, be possible to find examples of hard data if we considered beliefs expressed as 'this looks brown to me', or 'this seems to me brown'. An alternative analysis of knowledge such as this might enable us to find beliefs which possess more nearly the characteristics of non-inferred beliefs than those of Russell's examples. Whether or not the reader finds any value in the analysis that I shall briefly develop, it does at least, I feel, have the value of showing that other possibilities of distinguishing our knowledge into data and derived beliefs exist, so that Russell's difficulties need not require one to dismiss his basic distinction of knowledge into data and derived beliefs. One might say in analysing beliefs such as 'this is brown' that

[1] R. M. Chisholm, in *The Philosophy of Bertrand Russell*, pp. 434–5, discusses some criticisms of Ayer and Reichenbach to the same effect as that given above, that is, that propositions cannot be certain because they refer beyond the given. Chisholm attempts to answer this criticism as he thinks Russell might, but I cannot see that his answer really affects the point at issue.

what one means by the statement is that this is what *I* call brown, whether or not others agree. I believe, however, that usually if one says 'this is brown' he means it in the sense I have discussed in the preceding paragraphs. If one wants to refer to only his present sensible experience it would be clearer to say 'this is what I call brown', or 'this looks brown to me', or 'this seems brown to me'. What is desirable is a usage of language which will make clear that one is referring only to one's present experience, and not implying that agreeement with others is assumed. Then if one says 'this looks brown', he means that his experience of brown is the only test of the truth of his statement, that he does not intend to maintain that his sensible experience is of brown in the objective sense of implying the agreement of others when observing the same object.[1] In such a way it is possible to express beliefs about our perceptions in language that implies neither the objectivity nor the subjectivity of what is being perceived, in language that implies only the individual's bare experience. My own examples are only approximations to this end. In 'this looks brown', the evidence required for its truth is only my sensible experience, further sensations, or reports of the sensations of others are not required. The non-inferred quality, or 'certainty', of such beliefs does presuppose experience on the part of the individual concerned, since he must first himself know what he means by 'brown', and consequently beliefs certain for one individual may not in similar cases be certain for others. But this does not affect the general point of my discussion, since the certainty in question is determined by the evidence present to an individual for his own beliefs.

Analysing the certainty of beliefs from this other point of view does, I believe, lead to results different from those that

[1] There is a suggestion of this idea in *Philosophy*, pp. 206 ff., and *An Inquiry into Meaning and Truth*, p. 142, but I cannot see that Russell has developed it to any extent. The analysis I am contrasting to Russell's is perhaps closest to C. I. Lewis, and an excellent account to compare with Russell's is C. I. Lewis, *An Analysis of Knowledge and Valuation*, La Salle, Illinois, 1946, Book II, Chapter VII.

Russell obtains. Granting that the type of belief I have described fulfils Russell's criterion of certainty, it will be found possible to extend such an analysis to many apparently more complex cases. But the additional complexity, I believe, is only apparent, and these further cases are of the same kind as those I have already discussed. For example, 'this looks to me like a cow', 'this looks like a star', are certain on the same ground as 'this looks brown', that is, the only evidence required to establish them is the sensible experience the individual is having. Such statements do not maintain that the sensible experience is of an existent material object, but merely that it is of various sensible characteristics, normally those which such material objects possess. The sensible experience might be of a material object, it might be illusory, it might be a dream, but these characterizations are further statements about the experience for which other evidence than the original sensible experience is required. They would, following Russell, be inferences. The result of this analysis of knowledge is to find those statements certain which refer to our experience in the manner I have described, and statements that assert the reality, or unreality, of the objects perceived to be inferences from those certain statements. Russell's analysis, on the other hand, found those statements certain which are based on the smallest recognizable core of our experience, and all other empirical statements to be inferences based on them.

The alternative analysis of knowledge that I have been discussing could, I believe, be shown to avoid the difficulties Russell has encountered in determining the small core of experience that he finds is the only data, difficulties really based on the fact that his analysis is so formulated as to prevent him finding any non-inferred beliefs. But having modified the type of beliefs which can serve as data, it will be found that the problem of justifying (or replacing) the inferences to the material world has also been modified. For Russell, it was necessary to show that fragmentary percepts could be correlated with others in a group which would serve all the functions of

the usual material object. But with the new analysis the problem that we have is to determine how from 'this looks like a house' we can infer 'this *is* a house', or, on the other hand, 'I am dreaming of a house'. We are still, of course, faced with a difficult problem; but it is a different one from Russell's. We need no longer find how to 'construct' a house from restricted individual percepts.

I am not proposing to examine this other analysis of knowledge to see if by its means an answer to Russell's problem could be found; it might very well turn out that we could not use it to justify the inferences in the existence of material objects any more than Russell's can be used. I have advanced it primarily because it shows, accepting Russell's distinction of knowledge into data and derived beliefs, that other conceptions of what constitutes data are possible than Russell's, and consequently, different forms of the problem of justifying our belief in material objects. I believe that it is Russell's notion of hard data that is responsible for the direction his proposed solution takes, and it is consequently one of the crucial points of his philosophy.

The purpose for which constructions are utilized in Russell's works varies from time to time. We find them used for the epistemological purposes I have been discussing, to enable us to dispense with beliefs presumably inferred from data, at times they are employed to make it unnecessary to assume that there are entities such as material objects or points that are the denotation of symbols ordinarily assumed to stand for material objects or points. Constructions can then be used to replace 'inferred entities' with 'known' entities. We can view this latter use as the expansion of the former by the addition of the metaphysical belief that 'percepts' or 'appearances' are not only names that merely refer to the individual's sensible experience, but are existent 'particulars' or 'events', located in the brain of the percipient. Russell's problem is now how to eliminate (or justify) inferences from observed particulars, sense-data, to unobserved entities such as material objects.

Constructions are of no more success when applied to the

metaphysical aspects of his problem than they were when limited more to an epistemological problem. The reason, I believe, is the same, the employment of unperceived aspects, and what I believe is the impossibility of finding hard data by the procedure Russell uses. But before where I maintained that Russell's analysis was so formulated as to prevent us from finding any really non-inferred beliefs, that analysis now keeps us from any really 'known' entities. His epistemological problem is, I believe, a valid problem, and it is desirable to attempt to modify his analysis so that the difficulties he encounters can if possible be avoided. But I find it difficult to consider his metaphysical problem as valid a problem; for it depends largely on the metaphysical assumption of the reality of percepts, and their distinction from other entities existing outside the body. Russell as far as I know does not give any convincing argument for its adoption, his epistemological discussion as I have discussed earlier is itself no ground for it. It might be assumed for the same reason as Russell later in *Human Knowledge* assumed the view of the world which he believed science gives us, but this view again is not justified according to the standards which Russell apparently wishes to employ.

Whatever advantages this metaphysical assumption might have, it has the unfortunate result, combined with Russell's epistemological analysis, of leading him to a subjectivism which I feel he cannot avoid. His analysis of knowledge and his metaphysics lead him to maintain that the individual's knowledge is based upon fragmentary percepts which are in the individual's brain. Other particulars or events, those in other persons' brains or those outside any brain cannot be known directly, but only by inference from the percepts in the individual's own brain. It is only then by inference that the individual can know anything outside his own brain. The early use of constructions appears to assert that this is all that is necessary, although even here Russell included percepts in other brains besides the observer's. Later, Russell does maintain that the individual has knowledge of events outside his own brain, but such knowledge

is an inference based on the causal theory of perception, or the 'postulates' given in *Human Knowledge*. He finds it desirable for the individual to have knowledge beyond that of his own percepts, but he finds it is only possible to do so on the basis of postulates, or assumptions justified only on the ground that they do make that knowledge possible. It is basically only an assumption or a postulate that we can have empirical knowledge of anything beyond our own percepts.

The result that Russell has arrived at here is one which I do not feel to be inevitable, although I do not wish to explore alternative theories to the extent necessary to show this. His result is based, I believe, primarily on his conception of what constitutes non-inferred data and his metaphysical beliefs about percepts. The former could be modified in various ways, one of which I have suggested; the latter is an assumption which I find no compelling reason for Russell to adopt.

The emphasis throughout my discussion appears to be largely critical, but this should not lead the reader to a misapprehension of the value of Russell's work. There is after all little to be said besides praise for the parts that are beyond criticism, and I have assumed that the reader is sufficiently aware of the good points so that any further comment on my part would be unnecessary. The very fact of Russell's successful analysis of many difficult points in logic and philosophy, and the fact that he has at least been able to suggest a provocative analysis of further problems is sufficient recommendation for a careful reading of his works. My criticism, after all, could not have been made if Russell had not first formulated a careful analysis of his problem, one that has given him a secure position among the first rank of contemporary philosophers.

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